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APPLIED MECHANICS REVIEWS

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DISCONTINUOUS AUTOMATIC CONTROL

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The purpose of this article is a description of the development in this field in the last decade and of the present state of analysis and design of discontinuous automatic control processes. The reader should not expect to get a complete history. For that he may consult the introductory chapter of Ja. S. Tsypkin's book [1] which is available in the original Russian version (1955) and in a German translation (1958). The reader, also, will not get a complete list of references, but an easy access to them. An annotated collection of references up to 1955 can be found in the very interesting report by R. W. Bass [2].

Discontinuous automatic control is often called Relay Control, Bang-Bang Control, On-Off Control, and, finally, Contactor Control. A simple example for such a system is a second-order dynamic system in which an additional constant restoring force or the restoring force itself changes its sign at instances which are determined by performance requirements. These instances are not known in advance, but depend on the instantaneous behavior of the system.

Closely related to the contactor control problem is the control of systems with saturating elements (see, e.g., S. F. Schmidt's paper [3]). If saturation is reached fast, a first approximate analysis of the system behavior may be made on the basis of "relay control."

Recently Aizerman and Gantmacher [4] have investigated a discontinuous control in which, among other arrangements, the output of the nonlinear element is $\delta = N \operatorname{sgn} F + kF$. N and k are constants; in the simple "relay case" $k = 0$.

Discontinuous control is a nonlinear control, which is piecewise linear if a linear system (plant, vehicle) is controlled. Such a linear system may be of nth order and may be described by its transfer function obtained by application of the Laplace transformation to the system. This transfer function may have n poles and m zeros ($m < n$). Between two switching points the Laplace transform can be used for constructing the solution for the system with feedback contactor control. But since it does not provide more than a solution in steps (from one switching point to the next) it is not a fast method for handling this problem. However in many cases one will write down the transfer function of the system for mere convenience in communicating the particular features of the "plant" which shall be controlled.

An nth order linear system with contactor control is described by an ordinary differential equation of nth order or by a system of n ordinary differential equations of first order. It should be mentioned that most theoretical treatments of discontinuous control of nonlinear systems start with the assumption that the system is described by n nonlinear differential equations of first order.

The overall nonlinearity of contactor-controlled systems prevents superposition of solutions for obtaining a new solution; it also means that the character of the solution depends strongly on the initial conditions. A multiplication of the initial values by a constant factor may lead to a solution totally different from the one which was found for the original initial conditions. This is a peculiarity of nonlinear systems and it applies also to piecewise linear systems.

For quite a time the described features delayed the development of a convenient theory. The simplicity and ruggedness of the design, however, made engineers use discontinuous controls a long time before the theory reached its present advanced state. Occasionally there occurred serious stability troubles and, therefore, a theory was needed for analyzing the behavior of existing systems in addition to a theory for synthesizing new systems, which are supposed to perform a certain task.

Naturally it was tried to extend the frequency-response-stability criterion of Nyquist for investigating contactor controlled systems. Nearly every control engineer is familiar with R. J. Kochenburger's work. The basic assumption is that if a sinusoidal signal is impressed on the relay coil, the periodic square wave output of the relay can be replaced by its first harmonic. This assumption becomes more and more acceptable as the complexity (degree of the ordinary differential equation describing the system) of the system being controlled increases, because of the filtering action of these components. With this assumption, the action of the relay can be expressed in terms of a "describing function," which gives the amplitude (and in case of time delay in the relay, also a phase shift) of this first harmonic in terms of the amplitude of the impressed sinusoidal input. It was found, however, that occasionally important features of the controlled system could not be visualized in this way, because there is no due consideration of the transient response in this method. In Truxal's book [5] one finds a good historical background for this approach on p. 566 ff, and a critical discussion on p. 600 ff. Kochenburger's method, in general, facilitates the design of stable control systems. In particular he makes suggestions for favorable compensation networks, which determine the switching instances.

The investigation of the exact response to specific inputs sheds more light on the behavior of a system. As such were chosen

(a) initial disturbance only; input = 0 for $t > 0$,
(b) step input,
(c) follow up of a time-varying input of arbitrary or even random character.

In general the response is required to satisfy certain conditions and these, in turn, determine the switching function. Such conditions are zeroing of an initial disturbance or reaching a desired step in minimum time without overshoot or with minimum energy consumption. The latter condition usually needs more engineering specification than the first one; this fact is clearly reflected in a preference for the first problem by the large circle of mathematically interested authors. In case of an arbitrary input the mean square error for a desired time of operation may be required to be minimum or the local error is supposed never to exceed a given value, etc. A. T. Fuller [6] has discussed an error criterion of the form $\int_0^T \varepsilon^\nu \, dt$ with $\nu = 0, 1, 2 \ldots$ For $\nu = 0$ this criterion becomes equivalent to the minimum settling time criterion, when an initial disturbance has to be zeroed.

A very important point is that a sensible theory of discontinuous control has to include relay imperfections, which cause command and execution of switching not to coincide. The inclusion of these imperfections result in an understanding of the "chatter" phenomenon. Before this is discussed methods of describing the exact behavior of a system under contactor control will be described.

The most primitive method of description is the representation of the output $c(t)$ as a function of time based on a step-by-step computation. Each step comprises the time between two switchings. This method is rather laborious because the switching criteria are given by performance requirements and usually are not known in advance as a function of time. Ja. S. Tsypkin has developed a suitable way of handling the problem in this manner [1]. Another method has proved to be more powerful, at least for systems of order less than or equal to 4. One describes the motion of a dynamic system by considering the motion of a phase point in the phase space. The error function ε (difference between input and output; for zero input the error equals the negative output) and its derivatives are the coordinates of this phase space. For an nth order system one needs ε and the derivatives up to the $(n-1)$th. The moving point describes the "phase trajectory." If desired, the time can be marked as a parameter at this trajectory. Naturally the phase vector $(\varepsilon, \varepsilon', \varepsilon'', \ldots, \varepsilon^{(n-1)})$ can undergo a linear transformation and often new, more convenient, coordinates for describing the motion evolve.

A switching criterion $F(\varepsilon, \varepsilon', \varepsilon'', \ldots \varepsilon^{(n-1)}) = 0$, which is a linear function of the phase coordinates, represents a hyperplane in the phase space of nth order and a straight line in the phase plane used for visualizing the trajectory of a second-order system. This indicates that problems with a linear switching function can be handled easily. A rather detailed treatment of second, third and even fourth-order problems, whose transfer functions contain complex poles, is given in this author's monograph [7]. The coefficients of the derivatives in the function F will determine the type of solution. The desire is to choose these coefficients in such a way that one of the mentioned performance criteria is reasonably well satisfied. Rather early it became evident that the absolute minimum settling time, e.g., asks for a switching function which is much more complicated than a linear function of the error and its derivatives, particularly in the case that the transfer function of the controlled system has some complex poles.

For second-order systems without a zero and with real or complex poles the optimum problem (minimum settling time) was investigated rather early (see Kendall-Bogner [8], e.g., for real poles and Bushaw [9] for complex poles). Bogner and Kazda [10] and Bellmann, Glicksberg and Gross [11] showed that in an nth-order linear system with real poles only, an initial disturbance can be zeroed in minimum time by $(n-1)$ switchings. Such a simple rule does not exist for systems with complex or real and complex poles. Bushaw's important paper shows clearly that the magnitude of the initial disturbance determines the number of switchings in a second-order system with complex poles. The switching curve is a festoon curve (see Figs. 6 and 8 in ref. 9 or, better, Figs. 10.31 and 10.32 in ref. 12). If the plant has positive damping (stable plant), initial disturbances of any size can be successfully controlled; if the plant is unstable, contactor control can zero only disturbances of limited magnitude.

Higher-order systems with complex roots yielded solutions only in specific cases until a theory was developed which in its latest form allows determining the switching function as a function of time for any optimum criterion. This method has now become known as "Pontryagin's maximum principle"; ref. [13, 14, 15, 16]. Pontryagin and his associates showed that there exists a function H for each system (linear or nonlinear, represented by n first-order differential equations $\vec{dx}/dt = \vec{f}(\vec{x}; \vec{u})$ with $\vec{u}$ being the control vector) and each constraint, which has to become maximal at each instant in order to provide the desired switching function. This function H is constructed with the help of the solutions of a system of differential equations which are the adjoints to the given system. A rather simple analytic expression for the switching function F evolves. While Pontryagin, Boltjanski and Gamkrelidze studied optimum control for all autonomous systems with several control functions in Russia, some interesting papers appeared in this country (e.g., [11], [17], [18] and [19]), which contain a number of results which are now unified in Pontryagin's theory. In a later paper [20] LaSalle studied the minimum settling time problem for systems of the form $\vec{dx}/dt = A(t)\vec{x} + B(t)\vec{u}(t) + \vec{f}(t)$. [$A(t)$ and $B(t)$ are matrices and x, u, f are column vectors].

For a full understanding of the impact of Pontryagin's method on the design of optimum systems it is necessary to consider the switching function F in more detail. Its analytic expression contains unknown initial conditions of the adjoint system, which have to be adjusted to the initial conditions of the original system. Even if this task can be solved, the designer is not yet satisfied because he needs not $F(t)$, but $F(\varepsilon, \varepsilon' \ldots)$. Much effort therefore has to be put into getting some idea of the location of the switching points. Second-order systems with real or complex poles and nth-order systems with real poles are handled relatively easily. The difficulties arise if higher-order systems, with complex or real and complex poles, are investigated.

For third-order systems the location of the switching points in the phase space was investigated by I. Flügge-Lotz and Mih Yin [38] and H. A. Titus [39] for the criterion of minimum settling time. An iteration procedure has been given to determine the unknown initial conditions of the adjoint system. Principally this method could be used for setting up a computer program. However the last mentioned investigations show clearly that for pure initial error and zero initial derivatives, or pure step input, the requirements for minimum settling time are rather difficult to realize. Therefore one gets the idea that these optimum solutions merely give the designer an idea of what he can get at best. This idea may then let him evaluate if reaching the absolute optimum justifies a very large effort. In this light returning to a linear switching function and watching its performance as compared to the optimum switching function seems worthwhile. I. Flügge-Lotz and T. Ishi-

kawa ([21] and [22]) have investigated linear switching for third-order systems systematically, after A. A. Frederickson had studied particular third- and fourth-order systems [23].

One of the essential characteristics of linear switching is the chatter phenomenon. At an early date it was analytically described by Bilharz [24]. In a system with a perfect linear switching the trajectory may reach the switching surface under such an angle that the execution of the switching command would not change F from a positive to negative value or vice versa. That means F reaches zero, but does not change sign. Such a point is called a point of indecision; it is an "endpoint" (for more details see ref. [7]) for a perfect system, because after reaching this endpoint the motion is no longer defined by the idealized-systems equation. Only a delay in switching would make the motion "defined" again. In systems without zeros the delay (one type of imperfection, another is the threshold, which analytically has the same consequences) keeps the following motion close to the switching plane and starts a sequence of fast consecutive switchings, which means a "chatter." This chatter can be beneficial when it is controlled and leads to the desired end state. In some cases the nonlinear system behaves like a linear system in these chatter regions (see ref. [7], [21]). In systems with zeros, however, uncontrolled chatter may occur. The latter means a high frequency motion with an average motion equal to the motion without any control. It is obvious that this type of chatter should mostly be avoided. Uncontrolled chatter occurs [22], [23] when the system transfer function has zeros and when the number of zeros of the transfer function plus the number of zeros of the switching function exceed $(n-1)$ for the nth-order linear system. A restricted amount of uncontrolled chatter can decrease the peak-to-peak steady-state error for problems with initial disturbance or step input [22].

There are numerous papers dealing with the influence of a variety of "imperfections" on the performance of real systems. For second-order systems these investigations have led to extensive diagrams for the designer; as an example a paper by M. Rogers and G. Shapiro [25] is given.

All preceding remarks refer to a certain but very much used type of feedback contactor control: the output of the relay acts as input to the plant. There are other possibilities. Let us assume that the system to be controlled is described by an nth-order linear equation with constant coefficients. Then it is possible to influence these coefficients by feeding back positive or negative multiples of the output and its derivatives. The switching from positive to negative multiples is commanded by observing error criteria. That means one has to set up a "switching logic" for the process. Such a system was first suggested by I. Flügge-Lotz and W. S. Wunch [26], built by C. Taylor [27], and subsequently investigated by Minneapolis-Honeywell (see ref. [28], p. 385). Control systems belonging to this class but with different switching logic are discussed by Ostrovskij [29] and by Letov [30].

Thus far we have only briefly mentioned the problem of the stability of contactor controlled systems. Because of the nonlinearity of such systems it is obvious that Lyapunov's first method is applicable for investigating the stability of equilibrium points (stability in the small) and Lyapunov's second method for determining the region of stability in the phase space.

A brief introduction into Lyapunov's method for the designing engineer is given in the proceedings of a conference sponsored by the AIEE in 1960 [31]. For a more complete and advanced treatment the reader should consult W. Hahn's monograph [32], 1959. In the same year there appeared two papers by R. E. Kalman and J. E. Bertram, "Control system analysis" and "Design via the 'second method' of Lyapunov," which contain a discussion of the design of relay controls [33]. Both references [32] and [33] have a large bibliography, as has the survey paper by H. A. Antosiewiez [34].

Based on his own work and other investigation Tsypkin has discussed in ref. [1] in detail the stability of equilibrium points and he has given also the rules for determining if the points are stable. For plants with linear third-order transfer function these rules were established in a less rigorous way in [21], [22]. It should be mentioned that any investigation of "optimum control" includes the requirement of "stable control." However, imperfections in optimally controlled systems allow the occurrence of certain chatter types, unless a special design takes care of avoiding them.

The fact that in nonlinear systems auto-oscillations of finite amplitude can occur makes it necessary to determine the conditions for such oscillations in contactor control systems. For systems up to the third and even fourth order the visualization of such oscillation in the phase space is relatively easy. However in general an analytic method is preferred. After having found these cycles one has to determine whether these auto-oscillations (given by a limit cycle in the phase space) are stable or unstable. This stability investigation is based on Lyapunov's first method: One considers small deviations from the limit cycle and studies their decrease or increase with time. Tsypkin [1] again offers a fine discussion of this problem based on investigations of many authors. In any event the switching function should be chosen such that stable limit cycles with finite amplitude are avoided. For systems in which chatter occurs, usually the steady state is a chatter around the desired equilibrium point; however this type of high frequency oscillation can usually be tolerated, because its amplitude depends on the magnitudes of the imperfections of the system, and can be kept as small as desired.

The determination of the stability region of a control system with Lyapunov's second method requires the determination of a "Lyapunov function" V which is positive definite and whose total time derivative $\frac{dV}{dt} = \sum_i \frac{\partial V}{\partial x_i}\dot{x}_i = \sum_i \frac{\partial V}{\partial x_i} f_i(x, u)$ is negative definite in the region of stability (region described in the phase space with coordinates x_i). There is however to date no general method for finding such functions. It should be mentioned that it is possible to find more than one Lyapunov function for a given problem and that the regions of stability given by them differ in size; the Lyapunov function giving the largest region would have to be considered as the best one for the specific problem.

Lurje [35] has given such a function for systems of the form

$$\dot{\eta}_\kappa = \sum_{\alpha=1}^{n} b_{\kappa\alpha}\eta_\alpha + n_\kappa \xi \text{ with } \dot{\xi} = f(\sigma)$$

and

$$\sigma = \sum_{s=1}^{n} j_s \eta_s - r\xi,$$

where $f(\sigma)$ is a nonlinear function and satisfies the conditions $f(0) = 0$ and $\sigma f(\sigma) > 0$ for $\sigma \neq 0$. The $b_{\kappa\alpha}, n_\kappa$, j_s and r are constants. The function $f(\sigma)$ includes the possibility $f(\sigma) = N \operatorname{sgn} \sigma$ (N = constant), that is, relay problems are included.

Lurje has investigated systems up to fourth order; the difficulty of the investigation increases very much when one goes to higher than fourth-order systems. Other suggestions for constructing Lyapunov functions came from Lehnigk [36] and Ingwerson [37]. Lehnigk assumes for dV/dt a quadratic expression which contains weight coefficients; such coefficients allow modifications suitable for a particular problem. Ingwerson developed a procedure for nonlinear problems by

systematically generalizing known Lyapunov functions for linear problems.

The development of discontinuous automatic control shows that intentionally introduced nonlinearities can improve the behavior of dynamic systems and that it is worthwhile to overcome the analytic difficulties in the treatment of such systems because of their definite practical advantages. The interest of engineers and mathematicians is substantiated by the large number of papers on this topic appearing in rapid succession.

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Analytical Methods in Applied Mechanics

(See also Revs. 3960, 3970, 4039, 4053, 4095, 4115, 4133, 4158, 4184, 4234, 4246, 4256, 4306, 4324, 4535)

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Author demonstrates that many theorems of matrix algebra can be placed in table form. In this fashion, a vast amount of material is condensed in a small space. Idea of paper suggests that codification of similar type is applicable to other fields.

Y. Luke, USA

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From author's summary

3953. Tumarkin, S. A., Asymptotic solution of a linear non-homogeneous second-order differential equation with a transition point and its application to the computations of toroidal shells and propeller blades, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23,** 6, 1549–1565, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The transition point of the ordinary differential equation is a point where the dominant coefficient function has a simple zero. Following the method of Cherry [*Trans. Amer. Math. Soc.* **68** (1950)], author obtains a related differential equation whose coefficients match those in the given equation up to any specified order in the parameter involved. Solutions of the related equation can be expressed explicitly in terms of Airy functions and can be expected to furnish asymptotic approximations to solutions of the given equation. Representing a particular integral of the non-homogeneous related equation by variation of parameters and then integrating by parts, an expansion in powers of a parameter is obtained. The expansion is shown to furnish an asymptotic solution of the related equation and also to converge actually for a sufficiently regular nonhomogeneous term. The results are rather complicated as the various terms in the asymptotic expansion are functions of a variable which in turn is an expansion in powers of a parameter. The author does not relate the final results back to the given differential equation, nor does he discuss the error involved in replacing the given equation by a related equation. In an application of the method it would probably be impractical to retain more than one or two terms in the expansions.

Two applications are discussed very briefly, a toroidal shell expansion joint and a rotating propeller blade. The toroidal shell problem, as well as the general method of obtaining an asymptotic solution, may be compared with work of the reviewer [AMR 11(1958), Rev. 4917]. For a one-term approximation the results are the same. In principle, the results should be equivalent for higher-order approximations. However, the reviewer took a different point of view and there is not sufficient detail in the present paper to check the results. In the absence of internal pressure, both the author and the reviewer obtain the same simple formula for the spread of the expansion joint due to an axial load.

R. A. Clark, USA

3954. Bollermann, W., The inclusion of eigenvalues in the application of the maximum-minimum principle (in German), *ZAMM*, **40,** 7/8, 342–349, July/Aug. 1960.

In a self-adjoint, fully definite, ordinary differential system, Ritz-Galerkin method provides an approximation to the eigenvalue from above. For proper eigenvalue inclusion and hence accuracy estimation, however, approximation from below as well as from above must be provided. Author presents a method to approximate the eigenvalue from below by direct application of Courant's maximum-minimum principle of eigenvalues.

Reviewer believes that eigenvalue inclusion estimation was first given by G. Temple [*Proc. London Math. Soc.* (2) **29,** 257–280, 1928], and later extended by W. Kohn [*Phys. Rev.* (2) **71,** 902–904, 1947] and T. Kato [*J. Phys. Soc. Japan* **4,** p. 334, 1949], based on the principle of well-ordering of eigenvalues and the orthogonality requirement for iteration functions in relation to the eigenfunction. It is not difficult to establish the identity between Courant's maximum-minimum principle and the basis of Temple's solution. However, judging from reference list, author seems to be unaware of the existence of the work of Temple, Kohn and Kato.

H. S. Tan, USA

3955. Kreyszig, E., The treatment of elliptic partial differential equations by the methods of function theory (in German), *ZAMM* **40,** 7/8, 334–342, July/Aug. 1960.

The basic behavior of a second-order partial differential equation is determined by the relationship between highest derivative terms, i.e., the quadratic characteristic equation. Laplace and Poisson equations are specially simple forms of elliptic p.d.e., through which the harmonic (regular and singular) or potential function and functional theory of a complex variable have been correlated. Now the knowledge of this highly developed functional theory can be further utilized as a basis upon which solutions of elliptic p.d.e. in general can be studied. To this end two approaches are available: (1) By generalizing the definition of Cauchy-Riemann differential equations and so constructing a "pseudo-analytical" functional theory; or (2) by introducing an integral operator which transforms a complex analytic function into the solution of given p.d.e. Paper emphasizes the second approach, i.e., use of integral operator.

First part of paper discusses a special elliptic p.d.e., one for the stream function of steady two-dimensional compressible flow in hodograph plane. Solution to the d.e. can be constructed by expansion under two different requirements: (1) Separation of variables: d.e. leads to an infinite "trigonometric" series of velocity vector inclination angle, the amplitudes being hypergeometric functions of a velocity magnitude parameter. The solution must reduce to a harmonic function for corresponding incompressible flow. This constitutes the famous Chaplygin method. (2) By expanding in terms of harmonic functions of both hodograph variables: a recurrence differential relation is artificially constructed, leading eventually to an integral operator, i.e., a generating weight function under contour integral sign, which transforms the initial analytic term into final solution of the p.d.e. This is the well-known Bergman integral operator method. Of these two methods, the relative far-reaching generality of Bergman integral operator was emphasized.

Second part of paper deals with general form of elliptic p.d.e. with nonvanishing first-order derivatives, and gives a general method of constructing the integral operator of the solution, involving conjugate complex quantities as expected. General difficulties confronting actual construction of such integral operators are, however, pointed out.

Last part is an attempt to utilize the functional theory to handle three-dimensional Laplace equations. When there exists a preferred direction in the problem, complex variables can always be

introduced in the transversal. Bergman integral operators are constructed, and behavior of solutions qualitatively discussed. Possibility of extending this technique to situations involving spherical symmetry was indicated. H. S. Tan, USA

3956. Coleman, B. D., and Noll, W., An approximation theorem for functionals, with applications in continuum mechanics, AFOSR-TN-60-826 (Mellon Inst.), 33 pp., July 1960.

This report deals with continuous media that exhibit "memory" effects. As was recognized long ago by Boltzmann, the behavior of such systems may be described in terms of integral equations. If, however, the "memory" fades away sufficiently rapidly in time, the integral equation approximates to a differential equation of one order higher than that governing the appropriate limiting non-memory system, as is clear on intuitive grounds. To establish this on a rigorous basis is the authors' essential purpose. The arguments are conducted on a very abstract mathematical level throughout, and should be regarded as a contribution to mathematical analysis rather than to applied mathematics or mechanics. No references are made to J. G. Oldroyd's work in the same field.

M. A. Jaswon, England

Book—3957. Slonim, M. J., Sampling in a nutshell, New York, Simon and Schuster, 1960, xiii + 145 pp. $3.50.

This little book succeeds in giving humorous picture of nature of sampling methods and their use, without recourse to discussion of statistics. While not sufficient for a working knowledge of sampling theory, it will alert the engineer and manager to the powers and hazards of sampling for interpretation, control and evaluation. Author's decision to avoid the subject of variance makes the book more readable but calls for faith in the author instead of real insight at certain critical points.

The author uses either good luck or calculated risk when he compares accuracy of different sampling techniques by taking samples and comparing their means with the true mean.

E. W. Price, USA

3958. Gill, A., On a class of transportation problems, *J. Math. Phys.* **39**, 1, 76–82, Apr. 1960.

On a line n points called sources and numbered serially are situated. An object is transported from each source to one of another n points called destinations. A vehicle with one object capacity starts from one end, and after relocating all objects from the sources to the corresponding destinations reaches another end. Its schedule is specified by a sequence $(r_1, \ldots, r_n)$ at which the objects are picked up.

We express by e_{ij} the distance between the ith source and the jth destination. Our problem is to minimize the length of schedule in which the vehicle moves without object: $eor_1 + er_1 r_2 + \ldots + e r_n o$, where eor_1 is a distance between the start point and the r_1th source, and $er_n o$ the rth destination and the end point.

The minimizing procedure is as follows: Consider the sets of $n + 1$ off-diagonal elements of e_{ij}, each of which appears in a different row: $(eot_0, \ldots, ent_n)$. They do not always correspond to realizable schedules. We produce, in succession, all trial sets with $\Sigma eit_i = W, W + 1, \ldots$, where $W = \Sigma\, i \min eij$. Then the first trial set which proves compatible is the one which characterizes the minimum-length schedule.

The theory is applicable to the design of electronic circuits.

M. Sibuya, Japan

Computing Methods and Computers

(See also Revs. 3982, 4271, 4370, 4422)

3959. Albasiny, E. L., On the numerical solution of a cylindrical heat-conduction problem, *Quart. J. Mech. Appl. Math.* **13**, 3, 374–384, Aug. 1960.

The problem is $Z\dfrac{\partial\theta}{\partial t} = \dfrac{\partial^2\theta}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial\theta}{\partial r}$, $(0 < r < 1, t > 0)$ $\theta(r, 0) = 1$, $\dfrac{\partial\theta}{\partial r}(1, t) = -\theta(1, t)\, F(t)$, where Z is constant and $F(t)$ is an empirical function, which is the source of the difficulty in the problem.

Andrews [Cooling of a spinning thread-line, *Brit. J. Appl. Phys.* **10**, R. 39, 1959] gave an approximate analytical solution and stated that numerical integration would require excessive work because of the need for very small integration intervals. Author shows that numerical integration can be readily carried out on a digital computer, using the Crank-Nicolson implicit method. He introduces higher differences in the r direction and discusses their effect.

Author also considers the effect of the singularity due to the discontinuity of the gradient at $r = 1$, $t = 0$. He shows, analytically and by examples, that the resulting errors in the finite-difference solution decrease rapidly with time, so that no special starting procedure is needed unless accurate values of the solution for small values of t are required.

H. G. Landau, USA

3960. Dennis, S. C. R., Finite differences associated with second-order differential equations, *Quart. J. Mech. Appl. Math.* **13**, 4, 487–507, Nov. 1960.

Employing and generalizing a novel procedure due to D. N. de G. Allen and R. V. Southwell [*Quart. J. Mech. Appl. Math.* **8**, p. 129; AMR **9**(1956), Rev. 1504], author illustrates, primarily by examples with linear ordinary and partial differential equations, including eigenvalue problems, how one may obtain more accurate solutions to these problems through difference equations not involving larger numbers of points than customarily used.

The essence of the method when applied to linear ordinary differential equations is to consider them locally as differential equations with constant coefficients and derive a difference equation approximation to the original differential equation by eliminating the constant multipliers in the solution of the linear differential equation with constant coefficients in terms of the unknown functional values at neighboring points in the difference net. Linear partial differential equations are decomposed into ordinary differential equations to which the above technique is applied, the results of which are recombined to give a partial difference equation approximation to the partial differential equation. The experimental results show substantial improvements in accuracy over conventional methods.

This reviewer opines that the method has greater advantages in hand computation than in automatic computation, since in the latter a certain number of exponentials or trigonometric functions would have to be computed during the course of solution. It is possible that in automatic computation higher-order difference formulas could be used for the same amount of computation giving comparable accuracies. M. L. Juncosa, USA

3961. Sparacio, R., Solution by iteration of joint and panel equations (in Italian), *G. Gen. Civ.* **98**, 6, 460–468, June 1960.

After reviewing well-known slope deflection equations, author solves by iteration, following a procedure similar to the Cross method.

Four numerical examples are given.

A. J. Bignoli, Argentina

3962. Becker, S. J., An analysis of the yielded compound cylinder, *ASME Trans.* **83** B (*J. Engng. Industry*), 1, 43–49, Feb. 1961.

Paper continues the theory in "The theory of the ideal design of a compound cylinder," *ASME Trans.* **82** B, 136–142, 1960, by adding relationships for plasticity. Equations lead to pressures

and deformations at the elastic-plastic boundaries for the case of plane strain. A method of trial-and-error solution is suggested. Effects of compressibility are discussed. Locked-in stresses and permanent set after release of pressure are discussed. Application to a three-part cylinder illustrates method of analysis. Author plans to present the more difficult condition of generalized plane strain in later submissions. M. Holt, USA

3963. Krogdahl, W. S., Numerical solutions of the Van der Pol equation (in English), *ZAMP* **11**, 1, 59–63, 1960.

3964. Stroud, A. H., A bibliography on approximate integration, *Math. Comput.* **15**, 73, 52–80, Jan. 1961.

This is a valuable reference. There are some omissions. In private correspondence, the author advises of plan to issue supplements to fill the gaps and keep the material current.

Y. Luke, USA

3965. Emschermann, H. H., and Rohrbach, C., Direct electrical integration of closed calibration curves and their application for measuring the damping of metals and for indicating piston engines (in German), *ZVDI* **103**, 5, 169–176, Feb. 1961.

The methods so far employed for determining the integral of closed calibration curves, such as that of the damping work during a cycle in the case of cyclic stressed test pieces, or of the work content of the indicator diagrams of piston engines, are involved and very time-consuming, or they have a low frequency limit. By way of contrast, a new simple method enables the desired value to be indicated or recorded direct, during the measuring process, within a practically unlimited frequency range. This means that, for example, the damping of a material or of the work content of an indicator diagram where there are variable coefficients of influence (temperature, stress, etc. or load, speed and so forth) can be plotted direct. From authors' summary

3966. Baxter, D. C., Analog, digital and hybrid computers, *Nat. Res. Counc. Canada, Div. Mech. Engng. and Nat. Aero. Estab. Quart. Bull.* **4**, 41–54, Oct./Nov./Dec. 1960.

3967. Gaines, W. M., and Fischer, P. P., Terminology for functional characteristics of analog-to-digital converters, *Control Engng.* **8**, 2, 97–98, Feb. 1961.

Analogies

(See Revs. 4217, 4506)

Kinematics, Rigid Dynamics and Oscillations

(See also Revs. 4116, 4135, 4146, 4472, 4475, 4480, 4481, 4502, 4535, 4565)

Book—3968. Jenson, A., and Chenoweth, H. H., Applied engineering mechanics, 2nd ed., New York, McGraw-Hill Book Co., 1960, xiii + 409 pp. $6.50.

The interaction between calculus and mechanics produced the most characteristic and productive chapters in the history of natural sciences. It gives a strong feeling of claustrophobia and frustration to put mechanics in the strait jacket of a dim, hopeless, and calculusless world; nevertheless, authors chose to do this and not entirely without success. Book requires no mathematics beyond high school algebra and trigonometry. It is written as a textbook for college students in engineering (?) and architecture, for technical institutes, for industrial training courses and Armed Service programs.

The customary two parts, statistics and dynamics, are divided in 11 and 9 chapters respectively. Throughout the text the parallel, numerical-plus-graphical approach is emphasized and understanding is facilitated by a large number of very clear drawings. Dogmatic tendencies appear at some places and the student is urged to memorize for instance the two "laws of motion diagrams." The over 1100 problems assure that the student will acquire facility and technique to attack commonly occurring engineering problems.

The first edition appeared in 1947. The second edition, which is the subject of this review, is a revised and enlarged version of the first. Significant additions are belt friction, rolling resistance, hydrostatic loads, cables, noncentroidal rotation, impact, angular impulse-momentum relation, etc. V. G. Szebehely, USA

Book—3969. Parkus, H., Mechanics of solid bodies [Mechanik der Festen Körper], Wien, Springer-Verlag, 1960, viii + 264 pp. $6.90.

An introductory text on mechanics from a somewhat advanced standpoint. Subject matter is combination of general mechanics, elasticity theory, and mechanics of elastic structures. Breadth and depth of coverage in such a short book (264 pp.) is remarkable. Arrangement of material is orderly and logical. Some items included are kinematics of moving points and rigid bodies, relative motion, elementary mechanisms, bending-moment diagrams, plane trusses, the catenary, friction, ellipsoid of inertia, Euler's equations of the gyro, elementary vibration theory, bending of beams, Saint-Venant theory of torsion, shear center, vibrations of beams, stresses in rotating disks, symmetrical bending and vibrations of circular plates, helical springs, and theory of rotationally symmetric shells. Many examples are worked out. However, most teachers will disapprove of omission of problems for students to solve.

A deductive style, approaching an axiomatic treatment, is adopted. Basic laws are developed in vector form with complete generality. Although analytical rigor and clarity are exemplary, the empirical and philosophical aspects of mechanics, illustrated by classical experiments and the historical evolution of ideas, are sacrificed. There is very little discussion of fundamental concepts.

An original feature is preeminent position given to mechanics of deformable bodies. Rigid bodies and, indeed, all mechanical systems, are conceived to come within the scope of this theory. Accordingly, the concept of an isolated point mass is not introduced at all, although, naturally, in kinematics, motions of specific points are analyzed. Newton's law is expressed in the novel form, $\mathfrak{f} = \rho \mathfrak{b}$, where ρ is mass density, $\mathfrak{b}$ is acceleration, and $\mathfrak{f}$ is volume density of force in a body. By integration, this yields $F = m\mathfrak{b}$ for translating body.

In statics, vectorial nature of force is postulated. Couples and moments of forces are defined formally by means of vector products. Vanishing of resultant force and resultant moment are postulated as necessary conditions for equilibrium, although later these laws are derived by principle of virtual work. With aid of Gauss' theorem, volume integration of vectorial equilibrium equation of stress (including inertial force) leads to momentun principle for any mechanical system. Similarly, after preliminary vectorial multiplication of stress-equilibrium equation by the position vector, law of moment of momentum is derived. Hypothesis that stress tensor is symmetric replaces usual assumption that each internal force is collinear with its reaction. Kinetic and potential energy with related laws are introduced via dynamics of deformable bodies. D'Alembert's principle is expressed by equation $\int(\mathfrak{f} - \rho \mathfrak{b}) \cdot \delta \mathfrak{r}\, dV = 0$, which follows from Newton's law. It is interpreted to mean: "For any virtual displacement of a system from its instantaneous position, the total work of the internal, ex-

temal, and inertial forces is zero." Author should have called attention to continuity requirement of f, since, for example, this law does not determine equilibrium configurations of a system with Coulomb friction. Lagrange's equations are derived from D'Alembert's principle in usual way by introduction of generalized coordinates and application of chain rule of partial differentiation.

Elementary theory of elasticity is presented in conventional form with inclusion of quadratic terms in strain tensor. It precedes discussion of deformations of beams and other structural members. There are short chapters on strain-energy principles, thermal stresses, stability of equilibrium, the Ritz procedure, and impulse. Variational theory of mechanics is restricted mainly to systems with finite degrees of freedom, since calculus of variations is not introduced.

Printing, binding, and appearance of book are excellent. Text is to be highly recommended to those who desire reorganization and upgrading of teaching material in elementary mechanics.

H. L. Langhaar, India

3970. Freudenstein, F., and Sandor, G. N., On the Burmester points of a plane, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 41–49, Mar. 1961.

With a four-bar plane linkage it is possible, in general, to have a specific point on the coupler assume five arbitrary positions if the centers of rotation of the crank and lever are properly selected. These centers are called the Burmester points.

Paper is concerned with the location and properties of the Burmester points. In the first part, "Theory," an equation is obtained for the location of the points; geometric and algebraic properties of these points are deduced and special cases considered.

As intersections of third-order curves are involved in the location of the points, the algebra is complicated. The authors in the second part, "Computation," have developed a computer program for the IBM 650 which can be used for the determination of the points.

In part three, "Application," the analytical form of the theory is applied to the design of a four-bar linkage as a function generator, and a geared five-bar linkage to produce a prescribed path for the tracing point.

Reviewer feels the paper to be a valuable contribution to the applications of kinematic theory. The theory is not easy and the reader should have some acquaintance with it, such as is given in chapters 4 and 6 of "Kinematische Getriebesynthese" by Rudolf Beyer.

W. A. Wolfe, Canada

3971. Thorne, C. J., Some gyroscopic oscillations, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 57–66, Mar. 1961.

The case discussed is that of a gyroscope accelerated in a fixed direction, and precessed to bring the spin axis into coincidence with the direction of the acceleration. Though this is nowhere stated, this corresponds to the case of a vertical gyro with gravity erection. It would seem from the equations that the gyro is assumed to be pendulous, though again this is not stated, and symbols are used which are left undefined, except very partially by a rather inadequate diagram. Equations are nonlinear if second- and third-order terms are included, and are studied with the help of digital and analog computers. With the nonlinearities included, it is shown that unstable oscillations can result.

R. Hadekel, England

3972. Levitan, E. S., Forced oscillation of a spring-mass system having combined Coulomb and viscous damping, *J. Acoust. Soc. Amer.* **32**, 10, 1265–1269, Oct. 1960.

Considering a support-excited spring-mass system author solves its equation of motion through the application of a Fourier series to represent the Coulomb-friction force. The solution has been numerically evaluated for nearly all interesting combinations of the system parameters. The influence of such parameters as frequency-ratio, viscous damping ratio, friction-damping factor is illustrated by a large number of diagrams.

R. M. G. Muller, Indonesia

3973. Tsuda, K., On the vibration of a power-transmission system having angular clearances: Part 1, Harmonic, subharmonic, and ultraharmonic vibration of a system of single degree of freedom retarded by linear damping; Part 2, Approximate analysis of a system retarded by collision damping as well as velocity-proportional damping; Part 3, Experiment of a two-mass one-stiffness system retarded by collision damping as well as velocity-proportional damping; Part 4, Vibration excited by irregularities of gear teeth, analysis of an equivalent one-mass two-stiffness system reduced from a system with two degrees of freedom, *Bull. JSME* **2**, 6, 324–348, May 1959.

The series of papers deals with vibration problems occurring in most gear-driven systems. In particular, attention is focused upon the effects of gear or clutch clearances and variable gear stiffnesses.

Part 1 deals with the problem of forced vibrations of a one-dimensional system having combined linear elasticities. Under the assumption that a periodic solution exists, the analytical forms describing harmonic, subharmonic and ultraharmonic vibrations are combined by use of the boundary conditions of the stiffness functions. The stability of the resulting motion is concluded from the comparison of succeeding vibration amplitudes. An approximate solution for additional small velocity-proportional damping is given.

In Part 2 the effect of collision damping—characteristic for all vibratory systems with clearances—is considered. To initially arrive at the resulting motions the energy losses due to partially or fully plastic contacts are neglected. In a second step the analytical expressions so derived are used to determine the energy losses from an energy balance.

Part 3 forms the experimental section of the investigation. The test system is made up of an electrical dynamometer, a connecting shaft, a coupling allowing variable clearance, and a fan. Excitation is introduced by a piston-crank mechanism acting upon the fan. Due to the neglect of impulse losses in the analytical treatment (Part 2) the calculated collision velocities and torque amplitudes are about 20% above the measured values.

In Part 4 the method of Part 1 is used to investigate a one-dimensional system having variable clearance. The same procedure is applied to the system of constant clearance but periodically variable spring stiffness.

Almost no reference to the rather extensive literature on the subject is given.

A. Slibar, Germany

3974. Church, A. H., Simplified vibration analysis: Part 1, Mobility and impedance concepts; Part 2, Single-degree-of-freedom systems; Part 3, Two-degree-of-freedom systems; Part 4, Lumped multiple-mass systems; Part 5, Combination of subsystems; Part 6, Beams; Part 7, Frames, structures, and spring-mounted beams, *Mach. Design* **32**, 4, 130–139, Feb. 1960; **32**, 5, 134–141, Mar. 1960; **32**, 6, 180–176, Mar. 1960; **32**, 7, 116–122, Mar. 1960; **32**, 8, 179–184, Apr. 1960; **32**, 9, 141–150, Apr. 1960; **32**, 11, 135–142, May 1960.

Series of articles on mechanical mobility is of great interest for all who are concerned with practical computation of steady-state vibration problems of linear systems. Method is based on the knowledge of dynamical behavior of particular elements (mass, stiffness, damping) loaded by harmonically varying forces. The dynamical responses of particular elements, expressed as acceler-

ation, velocity or displacement, enable the construction of the solution of the whole system in a very clear and economical way without the use of differential equations. Method is suitable both for systems with concentrated or distributed parameters. The amount of work increases very considerably as more elements are added to the system.

Mechanical mobility is a new, very efficient method for practical evaluation of vibration problems. This series of papers can be recommended as a very useful help for a wide group of technicians concerned with vibration analysis.

V. Kopriva, Czechoslovakia

3975. Lebedev, A. A., The application of the method of "frozen coefficients" to the analysis of the steadiness of an irregular motion (in Russian), *Izv. Vyssh. Nebebn. Zavedenii. Aviats. Tekbn.* no. 1, 11-18, 1958; *Ref. Zh. Mekb.* no. 4, 1959, Rev. 3484.

The problem is investigated of defining the solutions $x_i(t)$ of the system of equations:

$$\frac{dx_i}{dt} = p_{i_1}(t)\, x_1 + \ldots + p_{in}(t)\, x_n + X_i + R_i \qquad [1]$$

$$(i = 1, \ldots, n)$$

over a finite time interval $t_1 \leqslant t \leqslant t_k$, by the quadratic forms:

$$v(x_1, \ldots, x_n, t) = \cdot \sum \beta_{ij}(t)\, x_i\, x_j$$

used in the capacity of Lyapunov functions. The theoretical analyses given in the paper repeat in great measure the considerations presented by the author in previous papers,—e.g., *Prikl. Mat. Mekb.* **20**, no. 5, 1957. The calculations necessary for the analysis of a concrete system of two equations are described.

The paper has been written more carelessly than is usual in mathematical papers. The problem investigated by this author has an extensive bibliography, but the paper quotes only three references.

E. N. Berezkin

Courtesy Referativnyi Zhurnal, USSR

3976. Mitropol'skii, Yu. O., Unsteady processes in some relaxatory oscillating systems (in Ukrainian), *Nauk. Zap. Kiivsk. In-ta* **16**, 16, 93-101, 1957; *Ref. Zh. Mekb.* no. 4, 1959, Rev. 3499.

A relaxatory oscillating system is investigated, described by equations of the form:

$$\frac{dx_k}{dt} = X_k(\tau, x_1, \ldots, x_n) + \epsilon\, X_k^*(\tau, \theta, x_1, \ldots x_n, \epsilon) \qquad [1]$$

$$(\tau = \varepsilon t,\ \varepsilon > 0);\ (k = 1, \ldots, n)$$

in which $X_k(\tau, \theta, x_1, \ldots, x_n)$ $(k = 1, \ldots, n)$ are periodic functions of θ with the period 2π; ϵ is a small parameter; $d\theta/dt = \nu(\tau) > 0$, for any $\tau \in [0, L]$. For this system of equations an approximate solution is found, depending on a single parameter.

M. E. Temchenko

Courtesy Referativnyi Zhurnal, USSR

3977. Strelkov, S. P., The application of Galerkin's method to problems of self-oscillation (in Russian), *Vestnik Mosk. In-ta, Ser. Matem., Mekban., Astron., Fiz., Khimii* no. 3, 51-55, 1957; *Ref. Zh. Mekb.* no. 4, 1959, Rev. 3503.

It is demonstrated that the application of B. G. Galerkin's method [Coll. Works, Vol. 1, p. 168; Akad. Nauk USSR 1952], in which the approximate expression (x) of the explicit function $y(x)$ by its inexplicit form $F(y) = 0$ is found as the sum of a number of given functions φ_i with weighted coefficients, determined from the condition of orthogonality of $F(\varphi)$ for each φ_i, for the determination of approximate, periodic solutions of nonlinear, differential equations, leads to the same, abbreviated equations as are obtained, in first approximation, by the Andronov-Poincaré or Van der-Pol methods or the method of slowly-changing amplitudes, Teodorchik's energetic method, etc. A more general formulation of the Galerkin method is given, on the basis of which the inaccuracy of the approximate solution according to Galerkin is evaluated, dispensing with the rigorous solution of the problem. The method is illustrated on the classic example of the "soft" excitation condition of a vacuum tube oscillator.

A. S. Alekseev

Courtesy Referativnyi Zhurnal, USSR

3978. Aminov, M. Sh., The steady rotation of a solid of variable mass about a stationary point (in Russian), *Izv. Vyssb. Nebebn. Zavedenii. Aviats. Tekbn.* no. 1, 3-10, 1958; *Ref. Zh. Mekb.* no. 4, 1959, Rev. 3487.

The equations of the problem are derived. The permanent rotation of a ponderous body about a vertical axis, and the steadiness of this motion, are examined under the following assumptions: the body remains continuously symmetrical; the principal axis of the fixed point remain continuously stationary with reference to the invariable part of the body; the fixed point remains on the axis of symmetry and the sum of the moments of all the forces excepting the force of gravity, with reference to the fixed point, is zero; the mass of the body, moments of inertia and coordinate of the center of mass, tend to become constant with time. The Lyapunov function is represented by a function becoming at the limit a linear linkage of the integrals analyzed by N. G. Chetayev for a body of constant mass. Since the derivative of this function is not zero, the sufficient conditions of steadiness, with the exception of the Mayersley condition, further incorporate the conditions of continuous negative sign of this derivative. The steadiness of the vertical axis for the case becoming in the limit the Kovalersky case, is analyzed in the same way.

G. K. Pozharitskii

Courtesy Referativnyi Zhurnal, USSR

3979. Carter, W. J., and Liu, F. C., Steady-state behavior of nonlinear dynamic vibration absorber, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 67-70, Mar. 1961.

A two-mass vibration system, main mass and dynamic absorber, is analyzed for the case of linear damping on main mass only and cubic nonlinearities on both springs. Analysis is based on one-term approximation and graphical procedure. Frequency ranges for unit amplification factor are given for particular case with hardening, linear and softening springs.

R. Plunkett, USA

3980. Baron, M. L., Response of nonlinearly supported cylindrical boundaries to shock waves, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 135-136 (Brief Notes), Mar. 1961.

The author solves the problem in the same way as his previous work. ["Response of nonlinearly supported spherical boundaries to shock waves," AMR **11** (1958), Rev. 1256].

First, he treats the response of the boundary of a cylindrical cavity with a linear pressure-displacement relation using transformation techniques. Secondly, using the same technique he studies the response of the cavity boundary to a suddenly applied unit step pressure which is radially symmetric. Making use of both results, he gets a nonlinear integral equation which can be evaluated numerically. Previous paper should be referred to for the analytical and numerical techniques.

More concrete result for practical application is desirable.

Y. Sato, Japan

Instrumentation and Automatic Control

(See also Revs. 3963, 3971, 4217)

3981. Ney, E. P., Maas, R. W., and Huch, W. F., The measurement of atmospheric temperature, *J. Meteorol.* **18**, 1, 60-80, Feb. 1961.

Authors combine theoretical, laboratory, and field studies to evaluate the behavior of thermometers in air. Special concern is with thermometers in form of small cylinders or wires used in the presence of visible and infra-red radiation at low atmospheric pressures and low rates of ventilation, but data is given for pressures from 0.1 to 1000 mb. Theoretical work assumes steady-state heat flow by conduction between air and thermometer; laboratory work confirms this assumption to be satisfactory except for very small wires. Field work confirms other results and shows in addition that heated or cooled boundary layers form around balloons, shrouds, etc., at high elevations so thermometers must be mounted to avoid these layers. Thin wires (approx 1-mil diameter) with high reflectivity to visible and infrared radiation show best promise of accurate performance over wide range of atmospheric pressures and radiation conditions. Graphs and tables are included. F. I. Badgley, USA

3982. Nicolo, R. G., Theoretical study and physical working of a logarithmic integrator (in French), *Automatisme* **5**, 7/8, 265–272, July/Aug. 1960.

Paper deals with an integrator circuit consisting of cascaded storage counters to provide an output voltage proportional to the common logarithmic rate of counting. To prove the accuracy of the apparatus statistical methods are applied. In the absence of the experimental verification, the validity of the several assumptions made is doubtful. It may be pointed out that the theoretical analysis up to the derivation of Eq. [19] is dealt with in the book "Pulse and digital circuits" by J. Millman & H. Taub, McGraw-Hill Book Co., 1956, pp. 346–53.

The paper apparently contains some misprints.

B. M. Belgaumkar, India

3983. Kazakevich, V. V., Reducing the influence of inertia in extremum regulation of n-th order objects, *Soviet Phys.-Doklady* **5**, 4, 662–665, Jan./Feb. 1961. (Translation of *Dokladl Akad. Nauk SSSR (N.S.)* **133**, 5, 1041–1044, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

A method is discussed in the present paper of reducing the influence of inertia of an object on the process of the extremum control as applicable to an *n*-th order object. The application of this method to objects of the first order was considered in previous paper by author in *Soviet Phys.-Doklady* **4**, 3, 578–581, Dec. 1959. [Translation of *Dokladl Akad. Nauk SSSR (N.S.)* **126**, 3, 517–520, May/June 1959.] [AMR **13**(1960), Rev. 5593.]

From author's summary

3984. Terano, T., Kinetic behavior of mono-tube-boilers, *Bull. JSME* **3**, 12, 540–546, Nov. 1960.

Author has undertaken a greatly needed examination of the behavior of a steam generator considered as a generalized system, limiting himself to a once-through type of forced circulation boiler. He starts with the five equations representing conservation of thermal energy in the boiler-tube material and in the steam-water mixture within the tubes, the heat transfer through the tube wall, conservation of mass in an element of tube length, and frictional pressure loss. Brief appendices give the transfer functions resulting from application of the usual Laplace transform methods to linearized forms of these differential equations. The results are displayed in the form of a very complex block diagram in Fig. 2, which is radically simplified without much discussion in Fig. 3. The open-loop response and closed-loop characteristics for several types of control system are shown in six additional figures. The lack of an adequate nomenclature and the severe limitation of discussion of the linearization steps and of the solutions obtained make it difficult to assess either the validity or the applicability of the results. M. A. Mayers, USA

3985. Haeussermann, W., and Kennel, H., A satellite motion simulator, *Astronautics* **5**, 12, 22–23, Dec. 1960.

3986. Lowrey, R. O., Space flight simulators, *Aero/Space Engng.* **19**, 10, 50–56, Oct. 1960.

3987. Dul'nev, G. N., Theory of control of heat for certain constructions of electronic systems (in Russian), Konvektivnyi i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 150–160.

3988. Dul'nev, G. N., Pokrovskaya, G. P., and Smirnov, A. I., Engineering method of calculating heat control of electronic equipment (in Russian), Konvektivnyi i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 161–175.

3989. MacKay, J. S., Approximate solution for rocket flight with linear-tangent thrust attitude control, *ARS J.* **30**, 11, 1091–1093, (Tech. Notes), Nov. 1960.

3990. Geissler, E. D., Problems in attitude stabilization of large guided missiles, *Aero/Space Engng.* **19**, 10, 24–29, 68, 70–72, Oct. 1960.

3991. Chapman, D. R., An analysis of the corridor and guidance requirements for supercircular entry into planetary atmospheres, NASA TR R-55, 47 pp., 1960.

See AMR **14** (1961), Rev. 1676.

Tables, Charts, Dictionaries, etc.

(See Revs. 3964, 4258)

Elasticity

(See also Revs. 3968, 3980, 4050, 4054, 4056, 4057, 4062, 4063, 4064, 4065, 4076, 4078, 4079, 4080, 4081, 4089, 4092, 4096, 4099, 4110, 4133, 4150, 4156, 4206, 4238)

3992. Lehmann, Th., Some observations on the description of processes in classical continuum mechanics (in German), *Ing.-Arch.* **29**, 5, 316–330, Sept. 1960.

In mechanics of continua, some concepts are most naturally defined with reference to points of space and others with reference to particles of the continuum. Present paper surveys the relations required for the change from the local to the material description. While the author admits that these relations are well-known, he claims that a survey of this kind is justified by the lack of clarity that is occasionally encountered in this field and by the fact that convected coordinates are not widely used.

W. Prager, USA

3993. Gurtin, M. E., and Sternberg, E., On the first boundary-value problem of linear elastostatics, *Arch. Rational Mech. Anal.* **6**, 3, 177–187, Nov. 1960.

Uniqueness theorems for the title problem (displacement w given at the boundary) are considered. A new proof of a theorem of Duffin and Noll [AMR **12**(1959), Rev. 5391] is presented; a new minimum principle is also established, which is based on a functional G introduced by Kelvin and which maintains validity when the strain energy U is not positive definite, provided that the "elastic constants" are such that the velocities of irrotational and equivoluminal waves are real. When the domain of integration

does not extend to infinity, the difference between G and U depends only on the given boundary values of w.

G. Capriz, England

3994. Paul, B., A simplified geometric proof of the reciprocal stress theorem, *Quart. Appl. Math.* **18,** 4, 395–396 (Notes), Jan. 1961.

3995. Frisch-Fay, R., A new approach to the analysis of the deflection of thin cantilevers, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 87–90, Mar. 1961.

The solution of a nonlinear second-order differential equation and the nonadaptability of the principle of superposition—every combination of loads must be solved on its own—are the main difficulties in the analysis of nonlinear bending. The paper proposes a new method for the calculation of large deflections. By applying the "principle of elastic similarity" author overcomes part of mentioned difficulties. Slender bars (cantilevers) with concentrated forces are reduced to one basic case, the strut problem.

O. Csellar, Hungary

3996. Ramakanth, J., Dislocation of the first order (in English), *ZAMM* **40,** 10/11, 512–514, Oct./Nov. 1960.

Author presents solutions for large deformation problems involving dislocations of the first order, i.e., for the problems of (1) a cylindrical tube with a small portion between two axial planes removed and the edges joined and (2) a cylindrical tube that is slit and the edges forced apart and constrained to lie in axial planes. Tube is assumed to be in a state of plane strain. Solutions are approximate for general case, but exact solutions are given for incompressible material and for some other special cases.

P. Seide, USA

3997. Kochetkov, A. M., Determination of pressure on elastic infinite plates between which lies a compressed elastic medium (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **29,** 92–97, 1960.

Using a stress function constructed with the aid of some Fourier integrals, author calculates the stress and strain state in the interior of an elastic layer subjected to a plane strain state. Assumption is made that the layer lies between two elastic or rigid rough plane plates. Between these plates and the layer a known constant tangential stress takes place. The total compressive stresses appearing in the middle plane of the layer are also known.

Finally, results for the normal stress appearing on the contact surfaces are given.

P. P. Teodorescu, Roumania

3998. Teodorescu, P. P., On the application of elementary calculation methods to the three-dimensional problems of the theory of elasticity (in German), *Rev. Mēcan. Appl.* **5,** 5, 649–667, 1960.

A solution in stresses is given for the problem of the three-dimensional elasticity with the aid of biharmonic polynomials. After an examination of the manner in which the biharmonic polynomials can be established the classical elementary solutions are re-found. The thick rectangular plate subjected to uniformly distributed load is then examined. Results are compared with those obtained in the strength of materials.

M. Soare, Roumania

3999. Deev, V. M., On forms of the general solution of the spatial problem of the theory of elasticity with the aid of harmonic functions, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 6, 1619–1622, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

By expressing the displacement vector as a sum of terms containing a harmonic vector R and the radius vector r in the form $u = \alpha R + \beta(\nabla R)\cdot r + \gamma r\cdot(\nabla R) + \delta r(\nabla\cdot R) + \varepsilon r^2(\nabla^2\cdot R)$ author determines the constants α, β, γ, δ from the equilibrium equations in displacements and also the stress expressions.

By various restrictive assumptions he obtains Papkovich-Neuber's solutions or those given by other authors, e.g., M. G. Slobodianskii, I. S. Arzhanykh, V. I. Blokh, as well as a series of additional solutions which result through the simultaneous consideration of all the terms considered in previous formulations.

The completeness conditions of the new solutions are not analyzed.

M. M. Misicu, Roumania

4000. Foppl, L., Conformal mapping of the plane elastic stress state (in German), *Forsch. Geb. Ing-Wes.* **26,** 6, 173–178, 1960.

Author relates the stress in a z-plane to a corresponding state of stress in a ζ-plane which arises by conformal mapping. As an illustration the stress in a plate, under all round tension, with an elliptic hole is deduced from the case of a circular hole. Apart from the necessity of knowing the solution for the circular hole, the method leads to not too easy calculations and appears to be less adaptable than the direct method given in the reviewer's "Plane elastic systems" (Springer).

L. M. Milne-Thomson, USA

4001. Mathys, R., Contribution to the solution of the strip problem (in German), Mitt. Inst. für Baustatik no. 34, 96 pp., 1960.

A semi-infinite plane disk is subjected to an equilibrated periodic loading in its plane along its straight boundary. A number of Airy stress functions (not necessarily orthogonal) are selected so that each of them should satisfy all but one of the boundary conditions along the loaded edge. The coefficients to be used in superimposing the stress functions are selected so that the thus calculated loading and the prescribed loading should be statically equivalent over short sections of the loaded edge. Assuming parabolic stress distributions over every pair of neighboring sections and replacing these by concentrated loads acting at the ends of the sections, the coefficients are readily calculated. The solution is extended to the following cases: (1) Semi-infinite strip loaded along its finite edge; (2) Rectangular strip; (3) skew parallelogram strip: (4) temperature stresses and strains in a rectangular strip. Numerical examples are provided for each application to permit comparisons with other methods for accuracy and to illustrate the amount of work involved.

G. Sved, Australia

4002. Wozniak, Cz., Solution of the two-dimensional problem for isostatic lines of constant curvature (in Polish), *Rozprawy Inz.* **8,** 3, 397–410, 1960.

The equations of plane stress are particularly simple if the curvilinear reference frame coincides with the lines of principal stresses (the isostatic curves), α_1 and α_2. General solution of the plane problem of elasticity is obtained in such coordinates, assuming that (a) the edges constitute also the lines of principal stress and (b) the curvature of these lines is constant. Thus many particular cases can be described in an instructive way, for regions bounded by circular arcs and loaded by concentrated forces.

The solutions obtained by the author coincide with those known from the literature and obtained in other ways often not so clear.

M. Sokolowski, Poland

4003. Kalandiia, A. I., On stresses in an elastic half-plane, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 6, 1615–1618, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Method, developed by Muskhelishvili, is used to find the stresses in an elastic half-plane when stresses at infinity are given. Interest of paper is primarily mathematical.

D. M. A. Leggett, England

4004. Malyshev, M. V., An approximate solution for the two-dimensional case in the theory of the limiting equilibrium of a friable medium (in Russian), *Inform. Materialy. Vses. Nauk-i. In-ta Vodosnabzh., Kanaliz., Gidrotekhn. Sooruzh. i Inzh. Gidrogeolog.* no. 7, 48 pp. 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4282.

An approximation method is presented for integrating the starting system of equations of the limiting equilibrium of a granular medium, in the conditions of the two-dimensional problem, founded on application of a linearizing approximation. The author writes the conditions of limiting equilibrium in nondimensional quantities as follows:

$$x(\sigma_x - \sigma_y)\sqrt{1+\psi^2} = (\sigma_x + \sigma_y + 2\operatorname{ctg}\rho)\sin\rho \qquad [1]$$

where

$$x = \operatorname{sign}(\sigma_x - \sigma_y), \qquad \psi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Next, the linear approximation is applied:

$$\sqrt{1+\psi^2} \approx \alpha + \beta\psi$$

which transforms Eq. [1] into

$$\sigma_x(x\alpha - \sin\rho) - \sigma_y(x\alpha + \sin\rho) + 2x\beta\tau_{xy} = 2\cos\rho$$

The coefficients α and β are completely determinate. The stress function $\varphi(x, y)$ is introduced which is linked with the stress components by the following relationships:

$$\sigma_x = \frac{\partial^2\varphi}{\partial y^2} + x, \qquad \sigma_y = \frac{\partial^2\varphi}{\partial x^2} + \varepsilon_0 x - s, \qquad \tau_{xy} = \frac{\partial^2\varphi}{\partial x\partial y}$$

The condition for limiting equilibrium then becomes transformed as follows:

$$b_1\frac{\partial^2\varphi}{\partial y^2} - b_2\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial x\partial y} = (b_2\varepsilon_0 - b_1)x \qquad [2]$$

where b_1, b_2, ε_0 are known constants. Thus the integration of the starting system of equations of the limiting equilibrium condition is reduced to finding the stress function φ, satisfying the linear equation [2] and the boundary conditions of the particular problem. Examples are given of some problems admitting of a rigorous solution.

A. I. Govyzdinov

Courtesy Referativnyi Zhurnal, USSR

4005. Theocaris, P. S., The stress distribution in a semi-infinite strip subjected to a concentrated load, *ASME Trans.* **81** E (*J. Appl. Mech.*), 3, 401–406, Sept. 1959.

Author presents an analytical solution of the problem of the stress distribution in a rectangular strip limited in transverse width, with a concentrated load at the center of the transverse edge in a longitudinal direction, the strip extending to infinity from the loaded transverse edge.

The problem is solved by using an analytical function defining the field of isostatics in the case where a normal load is applied at the boundary of a semi-infinite plate. The Schwartz-Christoffel transformation is used to map the field of isostatics to the corresponding field in the semi-infinite strip.

The results were checked by photoelastic tests using resin and plexiglass models and graphited paper for tracing the isostatics. The theoretical results correlate well with the photoelastic and electrical analogy methods. The theoretical solution presented here is not generally available in the literature. In the opinion of the reviewer it is a definite contribution.

E. G. Allen, USA

4006. Wittrick, W. H., Analysis of stress concentrations at reinforced holes in infinite sheets, *Aero. Quart.* **11**, 3, 233–247, Aug. 1960.

A hole of symmetric shape in an infinite sheet under pure tension or shear is considered with regard to the analysis of stress concentration. The boundary of the hole is compactly reinforced by a ring of constant cross-sectional area. It is assumed that the region outside can be conformally mapped onto the region outside the unit circle by a function in the form of a polynomial. Therefore the equations hold for different kinds of holes, such as triangles and rectangles with rounded corners; but the author only gives the preliminary theory without numerical results.

H. Neuber, Germany

4007. Szolagowski, F., An infinite disc with partly loaded circular hole (in English), *Bull. Acad. Polonaise Sci.* **8**, 8, 419–422, 1960.

Solution is incorrect. Author attempts to solve plane elasticity problem of infinite plate with circular hole whose boundary has uniform normal loading which subtends an angle α. Closed form stresses presented are independent of elastic constants. By a fundamental theorem of elasticity, non-self-equilibrated tractions on a boundary of a multiply-connected region imply either multiply-valued displacements or dependence of the stresses on the elastic constants. See Coker and Filon, "A treatise on photoelasticity," Cambridge, 1931, Chapter 4.

R. A. Eubanks, USA

4008. Tremmel, E., Expansion of the circular cylindrical cavity subject to local internal pressure (in German), *Ing.-Arch.* **29**, 5, 331–350, Sept. 1960.

An unbounded isotropic elastic mass with an infinitely long cylindrical cavity is subjected to localized internal pressure uniformly distributed along part of the cavity. Following C. J. Tranter [*Quart. Appl. Math.* **4**, p. 298, 1946], expressions are obtained for stresses and displacements in the form of infinite integrals, by means of the Love displacement function. Author then restricts himself to an approximate analytic evaluation of the integral involved in the radial displacement on the cavity surface, in particular in the case of concentrated loading. The method is to split the range of integration into parts, in each of which the integrand is replaced by a suitable approximating function. The analysis is very heavy and does not appear to offer any significant gain over direct numerical evaluation of the original integral. An illustrative example is worked in detail.

R. Hill, England

4009. Valentin, G., Thermal stresses in prismatic folded roofs (in German), *Österr. Ing.-Arch.* **14**, 3, 223–243, Oct. 1960.

First observing stresses arising when the temperatures outside and inside a prismatic folded roof are unequal, the publication goes on to derive equations for stresses and deflections. After treating folded roofs consisting of two parts paper deals separately with cases when slabs are simply supported or built in on their short ends at the plates at both ends of the roof. These two cases are described with examples. The results obtained show clearly the strain arising in folded roofs consisting of several parts.

K. Angervo, Finland

4010. Barrekette, E. S., Thermoelastic stresses in beams, *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 465–473, Sept. 1960.

Author discusses the behavior of elastic, free beams of arbitrary constant cross section under an arbitrary temperature distribution. The solution employed is an extension of a method of handling the two-dimensional thermal problem by means of a step-by-step solution due to Boley ["The determination of temperature, stresses, and deflections in two-dimensional thermo-elastic problems," *J. Aero. Sci.* **23**, pp. 67–75, 1956; AMR **9** (1956), Rev. 1393]. An analytical method of successive approximations for the solution of linear partial differential equations is applied to the

solution for the thermoelastic stresses. The method is applied to a general thermoelastic stress analysis of isotropic, homogeneous, simply connected, free beams. A solution is also given for the stresses and displacements in a right-cylindrical beam subjected to temperatures varying only with the axial coordinate.

R. L. Bisplinghoff, USA

4011. Smith, C. O., Thermal stresses and their significance in nuclear fuel elements, *J. Amer. Soc. Naval Engrs.* **72**, 3, 393–400, Aug. 1960.

The concept of the effect of temperature and temperature gradients in developing thermal stresses and strains is presented and discussed. Equations for the general case are developed and applied to the specific case of the circular cylinder (pipe). Temperature and thermal stress distributions for four different potential nuclear fuel elements are pictured and compared. Some conclusions of practical significance are drawn from this comparison.

From author's summary

4012. Trostel, R., Approximate calculation of thermal stresses with the aid of the variational principles of elasticity (in German), *Ing.-Arch.* **29**, 6, 388–409, 1960.

Following an expository section on variational principles in the classical theory of thermoelasticity, author considers in detail a generalization of the principle of complementary potential energy which yields a variational (quasi-static) characterization of the thermal stresses in the presence of given surface tractions, provided thermomechanical coupling is disregarded. This principle, in conjunction with the direct variational method of Ritz, is applied to the approximate solution of particular stationary and transient thermal-stress problems; the results thus obtained compare favorably with available exact solutions of the problems treated. In one of the examples dealt with, the standard Ritz procedure is modified in that the original problem is reduced to the solution of ordinary differential equations rather than to the inversion of a system of linear algebraic equations.

E. Sternberg, USA

4013. Piechocki, W., and Ignaczak, J., Thermal stresses due to a thermal inclusion in a circular ring and a spherical shell (in English), *Bull. Acad. Polonaise Sci.* **7**, 7/8, 419–424, 1959.

Analysis is first given for the plane stress distribution in a free annular disk with a localized inclusion at a different temperature; the stresses caused by a unit rise of temperature in a sector of the annulus are then determined by integration. A series solution is also given for the stresses in a thick spherical shell due to a unit rise of temperature in a conical sector.

E. H. Mansfield, England

4014. Zadoiyan, M. A., Temperature stresses in infinite concrete slabs in presence of creep (in Armenian), *Aikakan. SSR Gitutiummeri Akad. Tegekabir. Fiz.-Mat. Gitutiunneri Seria* **11**, 1, 27–46, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4277.

Applying the theory of an elastically-viscous body, an investigation is made of the temperature stresses in concrete slabs of infinite extent due to an unsteady heat flow, in the presence of creep of the concrete and the time-variability of its elastic deformation. The stress condition of an infinite concrete slab is investigated for the case of harmonic fluctuation of the ambient air temperature and cooling of the surface. Numerical examples are given. It is demonstrated that under the action of harmonic temperature fluctuations, the extreme values of $\sigma^*(t)$ (stresses in the slab in the presence of creep and an increase in the modulus of elastic strain) are nearly half those of $\sigma(t)$ (elastic solution). The solution of the problem of the temperature stresses in an infinite, concrete slab in the presence of symmetrical cooling of its surface leads to the important conclusion that the presence of creep introduces quantitative modifications of the elastic solution. In the presence of creep, the stresses $\sigma^*(t)$ change sign in the middle of the slab, against $\sigma(t)$, and, contrary to the elastic case, become tensile.

I. I. Ulitskii

Courtesy Referativnyi Zhurnal, USSR

4015. Riney, T. D., Residual thermoelastic stresses in bonded silicon wafers, *J. Appl. Phys.* **32**, 3, 454–460, Mar. 1961.

The residual thermoelastic stresses produced in a silicon strip when eutectic bonded to a gold-plated molybdenum block are investigated photoelastically using near infrared light and an image converter tube. This is the two-dimensional analog of a problem arising when silicon wafers are bonded to headers in semi-conductor devices. It is first shown that, with the proper orientation of the crystal, the specimen may be studied by the methods normally employed with isotropic birefringent materials. This simplifies the quantitative evaluation of the stresses. It is found that though the mean longitudinal stress near the center of the strip is compressive, a transverse tensile stress of greater magnitude occurs near the ends of the strip.

From author's summary

4016. Landau, H. G., and Zwicky, E. E., Jr., Transient and residual thermal stresses in an elastic-plastic cylinder, *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 481–488, Sept. 1960.

The authors derive equations for the stress rates in solid cylinders subject to transient temperature distributions. The material is assumed to be elastic-perfectly plastic obeying a Mises-Hencky temperature-dependent yield condition. The authors present a numerical procedure for integrating the equations and then apply their methods to a temperature distribution approximating a phase transformation and to a quenched cylinder. Special attention is given to the production of residual stresses.

R. L. Bisplinghoff, USA

4017. Piechocki, W., The problem of the non-steady state surface nucleus of thermoelastic strain in an infinite elastic body with a spherical cavity (in English), *Bull. Acad. Polonaise Sci. (IV)* **8**, 1, 1–4, 1960.

4018. McMillen, J. M., and Youngs, R. L., Stresses in drying lumber, *Southern Lumberman* **201**, 2513, 115–119, Dec. 1960.

Book—4019. Hearmon, R. F. S., An introduction to applied anisotropic elasticity, New York, Oxford University Press, 1961, viii + 136 pp. $5.60.

The book, intended primarily for applied mechanicists and physicists, presents the basic topics of anisotropic elasticity in a clear and concise manner. An up-to-date list of references, including Russian contributions in the field, is presented at the end of each chapter, and a general bibliography is given at the end of the book. The reviewer finds Chapters 6 and 7 of special interest. In Chapter 6 the problem of wave propagation in an anisotropic medium is discussed, and in Chapter 7 the problems of vibrations and elastic stability of anisotropic plates are presented.

E. Volterra, USA

4020. Brainin, E. I., On the question of anisotropy of strength in structural materials, *Soviet Phys.-Tech. Phys.* **5**, 8, p. 938 (Letters to the Editor), Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30**, 8, 1006–1007, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4021. Sobolevsky, V. M., The elastic stress condition of an anisotropic, hollow-sphere under the action of internal and external pressures and a radial heat flow (in Russian), *Doklady Akad. Nauk Bel SSR* **2**, 4, 147–158, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4106.

An investigation of the stress and strain condition of a hollow globe having spherical anisotropy, and transversely isotropic in the radial direction, acted upon by internal and external pressures; the solution is obtained in complete form. A solution is also presented for the case of the deformation of a sphere consisting of n concentric layers possessing spherical anisotropy, and transversely isotropic in the radial direction, on the assumption that the layers are rigidly interconnected.

V. A. Lomakin
Courtesy Referativnyi Zhurnal, USSR

4022. Kostromin, V. S., The solution of the plane problem in the theory of elasticity for the case of an anisotropic body (in Russian), *Sb. Trudl Voronezbok. Inzb.-Stroit. In-ta* no. 4, 59–62, 1958; *Ref. Zb. Mekb.* no. 4, 59–62, 1958; *Ref. Zb. Mekb.* no. 4, 1959, Rev. 4118.

It is demonstrated that the stress condition in the two-dimensional problem of the theory of elasticity for the case of an anisotropic body is, provided the anisotropy is rectilinear, independent of the elastic constants of:

$$\frac{\partial^2 \sigma x}{\partial y^2} = \frac{\partial^2 \sigma y}{\partial x^2} = 0;$$

and, if the anisotropy is cylindrical, of:

$$\frac{\partial \sigma_r}{\partial r} = \frac{\partial \sigma_\theta}{\partial r} = 0, \qquad \frac{\partial^2 \sigma_r}{\partial \Theta^2} = \frac{\partial^2 \sigma_\theta}{\partial r^2} = 0$$

These problems include, for instance, the case of pure bending of a rectilinearly-anisotropic, flat plate, the extra-central extension thereof, or extension by its own weight, the uniform extension in the radial direction of a cylindrically-anisotropic disk, etc.

A. S. Kosmodamianskii
Courtesy Referativnyi Zhurnal, USSR

4023. Nowacki, W., Steady-state stresses in an orthotropic cylinder and plate (in Polish), *Rozprawy Inz.* **8**, 3, 569–579, 1960.

A cylinder and a disk of a thermally and elastically orthotropic material are considered. Under the action of a steady-state sourceless temperature field (independent of the axial coordinate in the case of a cylinder), thermal stresses appear in an orthotropic body, in contrast to the isotropic case. These stresses vanish only if the material constants satisfy a number of additional conditions of special character. Similar phenomena appear for a plate clamped along the contour. The discussion is based on the Green's function representation of the state of stress.

M. Sokolowski, Poland

4024. Knops, R. J., The use of Poisson's ratio in studying certain non-homogeneous elastic inclusions (in English), *ZAMM* **40**, 12, 541–550, Dec. 1960.

Equations are developed for the perturbation in stress and deformation which occurs when inclusions of different Poisson's ratio, but with the same shear modulus, are inserted in an elastic medium. Their integration is shown to be reducible to the solution of a problem in potential theory and the results for an ellipsoidal inclusion are deduced. The analysis is extended to include the case of incompressible media and solutions to problems of cavities and of inclusions forced into cavities are indicated. An analogy with thermoelasticity is noted.

W. S. Hemp, USA

4025. Bhagavantam, S., and Chelam, E. V., Elastic behaviour of matter under very high pressures: Uniform compression, *Proc. Indian Acad. Sci.* (*A*) **52**, 1, 1–19, July, 1960.

Authors are interested in evaluating elastic constants of substances, already under large strains, as the first step in learning more about the interior of the earth. The method of approach is to first define "effective elastic constants" as the coefficients of the quadratic terms in the strain energy function corresponding to an infinitesimal strain superimposed over a finite strain, and then derive expressions for these constants by noting that the elastic energy of the infinitesimal deformation must be equal, after certain adjustments, to the difference in total energy of the system when subjected first to the finite strain and then to the finite plus infinitesimal strain considered as a single state of strain. The adjustments are necessary because of the change in volume of the element considered. Finite deformation theory is reviewed and then strain energy expressions are derived for finite strain in a material under hydrostatic pressure. Expressions for the effective elastic constants are then derived in terms of the hydrostatic pressure, and the constants of the stress-free state for a substance of cubic symmetry. Using such experimental results as are available it is shown that the effective elastic constants may increase up to a certain pressure and then start decreasing. Hence matter may be expected to behave in a very abnormal manner when pressure is very high. From this the authors deduce that it is quite possible for the material to become unstable under shearing stresses, and to thus become incapable of sustaining shear waves. This, it is implied, may be the explanation of why shear waves are not transmitted through the deep interior of the earth.

E. A. Ripperger, USA

4026. Karas, K., Strain and deformation in a rotating disk subject to axial torsional moments (in German), *Ing.-Arch.* **30**, 1, 63–76, Jan. 1961.

Author discusses the problem of disks of variable thickness subjected to the effect of forces due to inertial mass when considering the effect of inertia of blades. Shear stresses and deformations of the peripheral fibers of a disk are determined under the assumption of a constant distribution of shear stresses across the disk thickness. Author points out that the magnitude of shear stresses and deformations is directly proportional to angular acceleration. Solution is presented of a disk with constant thickness, disk of hyperbolic profile, constant-strength disk, conical disk, disk with exponential profile and disk with constant shear strength. Several numerical examples are given and curves of corresponding shear stresses and peripheral deformations are plotted in diagrams.

J. Valenta, Czechoslovakia

4027. Kuznetsov, A. A., Mechanical stresses in a stationary, rotating cylinder carrying a uniform electric current, *Soviet Phys.-Tech. Phys.* **5**, 5, 552–554, Nov. 1960. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30**, 5, 592–597, May 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

A solution is given to the problem of the mechanical stresses in a stationary, rotating cylinder carrying a uniform current, with the aid of the equations of elasticity theory and with allowance for the volume electromagnetic force.

From author's summary

4028. Dhaliwal, R. S., Stresses in certain thin plates rotating about an axis lying in their middle plane, *J. Franklin Inst.* **269**, 6, 463–473, June 1960.

For steady-state rotations, the problem becomes a two-dimensional problem of theory of elasticity. For its solution, the method of N. I. Muskhelishvili has been employed. Author considers plates with circular, dumbbell-shaped, cycloidal, and cogwheel-shaped boundaries. In each case the region of the plate is mapped on a unit circle. The paper contains some numerical results and diagrams.

M. P. Bieniek, USA

4029. Egorov, K. E., Contact problem for an elastic layer under the action of an eccentric vertical force on a circular rigid stamp, *Soviet Phys.-Doklady* **5**, 4, 890–894, Jan./Feb. 1961. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **133**, 4, 781–784, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4030. Kuznetsov, A. A., Mechanical stresses produced by the radial electromagnetic force in a multilayer coil wound with wire of rectangular cross section carrying a uniform current, *Soviet Phys.-Tech. Phys.* **5**, 5, 555–561, Nov. 1960. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* 30, 5, 592–597, May 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

The solution is given to the problem on the mechanical stresses in a multilayer coil by means of the equations from elasticity theory and with allowance for the electromagnetic force.

From author's summary

Viscoelasticity

(See Revs. 3956, 3992, 4014, 4041, 4112, 4113, 4147, 4203, 4204, 4212, 4221, 4257)

Plasticity

(See also Revs. 3962, 3992, 4016, 4056, 4080, 4158, 4186, 4192)

4031. Hillier, M. J., Stability of the hinge formed in an ideal rigid-plastic beam, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 133–134 (Brief Notes), Mar. 1961.

The hinge formed at yield in an ideal rigid-plastic beam is considered plastically stable if the moment of resistance increases after a small deformation, and unstable if the moment decreases. An initially curved beam of strain-hardening material and rectangular section is considered, and the change in moment of resistance due to a small change in curvature calculated. It is shown that, for a material which does not strain harden, the yield hinge is unstable if the initial curvature and the change in curvature are of the same sign. However, the initial rate of change of moment of resistance is small. The change of yield moment of a straight beam is initially zero. The minimum initial hardening rate required for stability of a curved beam is calculated.

From author's summary by O. Csellar, Hungary

4032. Nemeti, L., On the incipient plastic state in tubes subject to internal pressure (in German), *ZAMM* **40**, 12, 551–557, Dec. 1960.

A design criterion based on maximum permanent deformation is used to determine the limiting pressure in a circular cylindrical tube. Analysis is based on the Hencky total strain equations and is therefore limited to monotonically increasing loading. An equivalent yield stress is derived which is a linear function of the prescribed permanent strain. This result seems to be of limited practical interest referring only to permanent strains small compared with the elastic strains. J. Hult, Sweden

4033. Higginson, G. R., The theoretical strength of band-reinforced pressure vessels, *J. Mech. Engng. Sci.* **2**, 4, 298–301, Dec. 1960.

A cylindrical pressure vessel is reinforced by circumferential rings and subjected to internal pressure. Upper bounds on the yield-point load are obtained on the assumption that both the vessel and the rings are perfectly plastic. The bounds are based on simple assumed velocity distributions and are minimized with respect to their parameters. Corresponding lower bounds are not obtained.

The resulting dimensionless yield-point loads ($P = pa/Yb$) are found to depend on two dimensionless parameters

$$H = \pi^2 ab/2l^2 \text{ and } \gamma = (Y_r/Y)(tc/lb)$$

where a, b, l, and Y are the radius, thickness, yield stress, and length of the vessel, and tc and Y_r are the area and yield stress of the ring. If $H \geq 2$, the strength of an open cylinder is almost as great as if the same total material were used in an unreinforced cylinder; for closed cylinders the rings do not seem to be as efficient in providing yield-point load strength.

P. G. Hodge, Jr., USA

4034. Datsko, J., and Yang, C. T., Correlation of bendability of materials with their tensile properties, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 309–314, Nov. 1960.

The authors give a method of analysis for predicting the critical "bend radius" for a specimen subjected to bending from the uniaxial tension percentage reduction in area. Experimental data were obtained to determine the validity of the prediction. The experimental and theoretical results were found to be in good agreement. This makes it possible to determine the minimum bend radius for a specific material from a standard tension test. It should be noted that the analysis is restricted to materials with the same properties in tension and compression.

J. Marin, USA

4035. Roderick, J. W., The elasto-plastic analysis of two experimental portal frames, *Struct. Engr.* **38**, 8, 245–254, Aug. 1960.

Using stress-strain curve representative of average results from flexural tests, true moment-curvature relations, including effect of knee from proportional limit to yield point and effect of strain-hardening, were derived. Load-deformation relations for a pin-ended rectangular frame subjected to horizontal and vertical loads were calculated, first neglecting then considering additional moments due to frame sidesway. Neglecting them, strain-hardening effect gave load increasing with deflection above observed collapse load and above failure load predicted by simple plastic theory. Considering additional moments, collapse was predicted at slightly less than observed collapse load. Load-deformation relations for a fixed-ended frame gave little difference between results neglecting or considering sidesway, and no collapse was predicted or observed. Reduction in plastic capacity due to axial load was not considered. Method developed gives better picture of behavior close to collapse load predicted by simple plastic theory, and author points out that actual collapse can occur, at times, at loads significantly lower than predicted by simple plastic theory.

J. Chinn, USA

4036. Pian, T. H. H., Yield conditions of plates and shells by Mises-Hencky criterion, AFOSR TN 60–608 (Mass. Inst. Technol., Aeroelastic & Struct. Res. Lab. TR 76-3), 13 pp., June 1960.

Making use of the Mises-Hencky yield criterion, author derives expressions for the yield conditions for a thin plate or shell under combined bending moments and normal forces without twisting moments and shear forces. The yield conditions obtained are expressed in terms of three parameters and do not admit an easy application. For several limiting cases such as pure bending, uniaxial bending and membrane stresses and bending with one of the curvature rates equal to zero, the parameters can be eliminated and simple yield conditions result. Some results are compared with those obtained on using the Tresca yield condition.

Z. Karni, Israel

4037. Makky, S. M., Plastic flow and fracture in round bars under pure torsion, *J. Math. Mech.* **10**, 2, 199–221, Mar. 1961.

Consideration of the dynamics of an elastic, perfectly plastic material leads to the conclusion that discontinuities should propa-

gate on a helicoidal surface making an angle of 45° with the generators of the surface of the bar. Reviewer points out that singularities and fractures normal to the axis do indeed exist, perhaps due to strain softening. See F. P. Lybalko and M. V. Yakutovich, *Fizika Metallov i Metallovedenie*, **4**, 3, 450–454, 1957.

F. A. McClintock, USA

4038. Kuzniecow, A. I., The problem of a nonhomogeneous plastic layer (in Russian), *Arch. Mech. Stos.* **12**, 2, 163–172, 1960.

The paper is composed of two parts loosely connected with each other. Each part contains generalization of known solutions of the theory of plasticity for a homogeneous body to bodies of which the yield point is variable with certain coordinates. In the first part the Prandtl problem of a plastic body in a plane state of strain compressed between two parallel plates is solved assuming that the yield point $k(y)$ is variable along the y-coordinate normal to the plate. Two equations of equilibrium and the yield condition in which the known function $k(y)$ appears constitute a system of three equations with three unknowns.

This system is solved assuming that the shear stress depends on the y-coordinate only. The integration constants are determined from the boundary conditions and from the equilibrium condition of a strip of length x. Also, the flow velocity distribution is obtained. The result does not satisfy all the boundary conditions and, similarly to the Prandtl solution, approaches the accurate solution only in cross sections sufficiently distant from the end edges of the compressed strip.

The particular case is considered where the yield point $k(y)$ varies in a linear manner.

In the second part of the paper the Ilyushin problem is considered for a thin layer with a definite boundary compressed between two parallel plates assuming that the yield point is a function of all three coordinates. This nonhomogeneity may result from a nonuniform temperature field during hot-forming of the material. It is shown that the Ilyushin sand-hill analogy for a homogeneous body cannot be applied. However, there is a certain analogy with the problem of torsion of a nonhomogeneous plastic bar studied in one of the author's papers. A numerical example is given for the compression of a rectangular layer with the yield point variable along the thickness and the longer edge.

W. Szczepinski, Poland

4039. Czyzak, S. J., Bow, N., and Payne, H., On the tensile stress-strain relation and the Bauschinger effect for polycrystalline materials from Taylor's model, *J. Mech. Phys. Solids* **9**, 1, 63–66, Feb. 1961.

Taylor's [*J. Inst. Metals* **62**, p. 307, 1938] theoretical model for calculating the stress-strain relation for a crystalline aggregate, as extended by Lin [*J. Mech. Phys. Solids* **5**, p. 143, 1957; AMR **10**(1957), Rev. 3640] is used to predict the behavior of a polycrystalline material subjected to plastic strain, first in simple tension, and then in simple compression.

Experimental results show that better agreement with the extended theory is attained when the critical shear stress τ_c is taken to increase with λ rather than be constant.

D. Kececioglu, USA

4040. Baranovski, M. A., and Yurkshtovich, N. A., The physico-chemical theory of plasticity (in Russian), *Sb. Nauchn. Tr. Belorussk. Politekhn. In-ta* no. 66, 117–136, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4249.

A survey of the labors of S. I. Gubkin and his collaborators. The physico-chemical theory of the plasticity of solid bodies deals with the mechanism of plastic deformation of such bodies in relation to their chemical composition and physical structure. The researches were conducted in three directions: (1) Investigation of the capacity for deformation of metals and development of a theory of deformability; (2) discovery of new methods for investigating the plastic behavior of metals; (3) researches in the direction of intensifying existing, and developing new, technological processes for the working of metals by pressure.

N. I. Malinin

Courtesy Referativnyi Zhurnal, USSR

4041. Reiner, M., Plastic yielding in anelasticity, *J. Mech. Phys. Solids* **8**, 4, 255–261, Nov. 1960.

Material is considered in which yielding occurs after creep. The notion of the Von Mises criterion is extended to allow for the amount and rate of creep using only that part of the distortional stress-work which is conserved as elastic energy. The results are qualitatively in accordance with experience. More precise comparison with experiments would be of considerable interest.

W. M. Shepherd, England

4042. Ivlev, D. D., On the properties of the relations of the law of anisotropic hardening of plastic material, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **24**, 1, 191–194, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Paper is a theoretical discussion of the anisotropic hardening law proposed by Shield and Ziegler [AMR **12**(1959), Rev. 2824]. Author shows that the equations are hyperbolic so that a slip-line field of approach can be used for such materials. He establishes the necessary equations for plane strain conditions, and for problems in three dimensions (with Tresca yield criterion).

J. M. Alexander, England

4043. Osipov, V. O., Methodology of the experimental investigation of the relaxation of residual stresses and their summation with load stresses, and some results of its application (in Russian), *Trudi Mosk. In-ta Inzh. Zh.-d. Transp.* no. 101, 167–199, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4614.

A new method is suggested for determining residual stresses, termed the "aperture method," which consists in the following: A hole is drilled in the sample, and a strain gauge placed nearby. Loads are applied to, and removed from, the test sample. The resulting residual stresses are calculated from the strain gauge readings before and after loading by the equations of the theory of elasticity. Instructions for using the method are given. The results of application of this new method are sufficiently accurate for the practical evaluation of residual stresses.

V. S. Namestnikov

Courtesy Referativnyi Zhurnal, USSR

4044. Sansome, D. H., and Lloyd, H. K., An investigation of the Pilger process by a plane strain analogue, *J. Mech. Engng. Sci.* **2**, 4, 359–381, Dec. 1960.

Subject refers to process of hot-rolling seamless tubes. Paper describes an analogous plane strain experiment on lead together with application of plastic theory of indentation by flat die. Experimentally determined load-torque curves are presented. Authors conclude parabolic cam has most desirable profile.

G. W. Housner, USA

4045. Davis, E. A., Creep rupture tests for design of high-pressure steam equipment, *ASME Trans.* **82** D (*J. Basic Engng.*), 2, 453–461, June 1960.

The results of an approximate creep analysis of heavy-walled cylinders agree fairly well with the results of tests under internal pressure and axial load.

Although the largest strain occurred at the hole, failure began at the outside surface where the hydrostatic component of the stress is largest.

J. D. Lubahn, USA

Rods, Beams and Strings

(See also Revs. 3968, 3974, 3995, 4005, 4010, 4016, 4031, 4103, 4105, 4116, 4117, 4118, 4121, 4135, 4147)

4046. Strel'bitskaya, O. I., The determination of the limiting load on bars in bending and torsion (in Ukrainian), *Dopovidi Akad. Nauk URSR* no. 6, 543–437, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4257.

The limiting conditions are examined for double tee and channel section beams, in simultaneous bending and torsion. Formulas are suggested for determining the limiting load. Applying the equation obtained earlier, linking all the force factors (M_x, Q, B, H and M_w) and considering that, in the cross sections of single span beams in which the bending moment M_x and the binary moment B are greatest, the torsional moment M in free torsion is zero, as well as assuming that the binary moment $B = M_x e y_1$, author obtains a relationship between the bending moment M_x and the transverse force Q in the form:

$$\frac{M_x^2}{\sigma T^2} R_\gamma + \frac{Q^2}{\sigma T^2} S = 1$$

In the equation presented, the coefficients R_λ and S are determined by the dimensions and profile form of the cross section and the coefficients γ and γ_1, depending on the loading pattern and mode of end constraint of the beam. A table of equations is presented for calculating the limiting load in six cases of single-span (simply supported) beams, loaded either by a concentrated force or by a distributed force. For the same cases, expressions are given for the limiting load values in pure bending (in the absence of torsion) and in pure torsion (in the absence of bending).

E. A. Raevskaya

Courtesy Referativnyi Zhurnal, USSR

4047. Kholontsev, V. V., A method of analysis for beams with perforated walls (in Russian), *Nauchn. Trudi Odessk. In-ta Inzh. Morsk. Flota* no. 16, 112–130, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4386.

The method is applicable to determination of the normal and shearing stresses with, in the general case, an irregular series of perforations along a wall. Analysis is performed for the presence of bending distortion of the beam in the elastic region. Local deformation and loss of stability of the beam elements are not considered. The solution of the resulting differential equations has been developed to a working basis for the particular cases of a regular series of holes, symmetrical and unsymmetrical with reference to the center line of the beam. In the form as presented, these equations are somewhat cumbersome to use; auxiliary charts are not furnished, and their absence makes proper evaluation of the method somewhat difficult. It is indicated that the method has been confirmed by experiments made by the author [title source no. 13, 235–251, 1957].

S. Ya. Makarov

Courtesy Referativnyi Zhurnal, USSR

4048. Zakharov, K. V., The bending of orthotropic beams by a shearing load (in Russian), *Trudi Leningrad. Politekhn. In-ta* no. 196, 73–87, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4382.

Orthotropic beams are examined, the cross section of which is a narrow rectangle, for the case of action of shearing forces of differing intensity, T_1 and T_2, uniformly distributed along the longitudinal edges. Two cases of end constraint are considered: the cantilever beam, and the beam on two supports. The solution of the fundamental differential equation for the stress function is sought in the form of the sum of the whole polynomials from the second to the fourth order inclusive, with indeterminate coefficients, which are found from the boundary conditions and the conditions of equilibrium for the cut-off portion of the beam. Expressions are set up for the stresses, which show that anisotropy of the elastic properties of the material has an influence in the presence of lack of coincidence between the beam axis and the axis of elastic symmetry. If these axes coincide, however, the stress condition is no different from that of an isotropic beam. Anisotropy of the material influences only the state of deformation of the beam.

E. F. Burmistrov

Courtesy Referativnyi Zhurnal, USSR

4049. Portayev, L. P., The analysis of beams on an anisotropic soil foundation (in Russian), *Trudi Mosk. In-ta Inzh. Gor. Str-va* no. 8, 106–119, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4389.

A model of an elastic foundation having varying values of the modulus of elasticity in depth E_z and horizontally, E_r, is investigated. A linear relationship is assumed between the stresses and the strains, and no regions of plastic deformation exist. The stresses and displacements are determined by the equations of the elastic theory for anisotropic bodies. Any suitable method of stress analysis for an isotropic semispace is suggested to be used for calculating the beam, slightly modifying the coefficients of the fundamental equations. A numerical example is given, and comparisons are made.

P. I. Klubin

Courtesy Referativnyi Zhurnal, USSR

4050. Tyagunov, I. A., Transverse bending of a cantilever type prismatic bar having a cross section in the form of an isosceles triangle and loaded in the plane perpendicular to the plane of symmetry (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **26**, 270–272, 1958.

Author gives fairly simple expressions for the two shear stress components in a cantilever bar and shows that they fulfill the equilibrium and compatibility equations and the boundary conditions. Stress components correspond generally to a combination of transverse bending and torsion and the angle of rotation of the bar becomes zero only if the value of Poisson's ratio is chosen in a definite manner depending on the vertex angle. This value is 1/2 for the equilateral triangle and in the case of this section the stress components remain finite only if that value is taken for Poisson's ratio.

A. Kuhelj, Yugoslavia

Plates, Shells and Membranes

(See also Revs. 3953, 3962, 3998, 4000, 4001, 4003, 4009, 4013, 4026, 4033, 4103, 4107, 4108, 4110, 4111, 4115, 4126, 4131, 4151, 4152, 4157, 4244, 4247)

4051. Hooker, W., and Protter, M. H., Bounds for the first eigenvalue of a rhombic membrane, *J. Math. Phys.* **39**, 1, 18–34, Apr. 1960.

The method, given by Picard and later used by Boggio, for the determination of a lower bound for the first eigenvalue of a membrane is well known. Consider the membrane equation $u_{xx} + u_{yy} + \lambda_1^2 u = 0$ for a domain D with boundary Γ and put $u = 0$ in Γ.

When the domain is a rhombus, Polya and Szego, by employing Steiner symmetrisation, determined the lower bound of the first eigenvalue: $\lambda_1 \geqslant \dfrac{\pi}{d} \dfrac{(1 + \sin^2\beta)^{1/2}}{\cos \frac{1}{2}\beta}$ where β is the acute angle opening and d the length of the shorter diagonal of the rhombus. Author obtains against $\overline{\lambda}_1 \geqslant \dfrac{\pi}{d} \dfrac{(1 + \sin\beta)^{1/2}}{\cos \frac{1}{2}\beta} = \dfrac{\pi}{d}(1 + \operatorname{tg} \dfrac{\beta}{2})$ and as $\overline{\lambda}_1 \geqslant \lambda_1$, $\overline{\lambda}_1$ results to be a better limitation.

The value of $\bar{\lambda}_1$ is obtained by using oblique coordinates, carrying back the problem to solution of two Riccati equations.

Afterwards author shows also that in rectangular coordinates problem may be solved using Riccati's equations, but the lower bound thus obtained for λ_1 is less satisfactory than $\bar{\lambda}_1$.

At the end author determines an upper bound for thin and for nearly square rhombuses and presents some numerical tables and graphs.

G. Supino, Italy

4052. Weber, C., Infinite cantilever plate under concentrated load (in German), *ZAMM* **40**, 12, 558–565, Dec. 1960.

The abstract states: A strip of plate infinitely long is clamped on one of its sides while the other side is free. A concentrated force acts at an arbitrary point. A solution is deduced which is expressed by potential functions with singularities.

Approximate values for maximum support moment are given. More accurate results may be found in the book "Theory of plates and shells," 2nd ed., pg. 211 and 337, 1959 by S. Timoshenko and S. Woinowsky-Krieger.

E. H. Dill, USA

4053. Hanuska, A., Application of the method of perturbations for the investigation of the influence of Poisson's ratio on the state of stress in thin plates (in German), *ZAMM* **40**, 12, 570–571, Dec. 1960.

Kirchhoff's theory of thin elastic plates is used. The deflection is a nonlinear function of Poisson's ratio ν. This function may be a complicated one, especially if the boundary conditions are not independent of ν. Using the differential equation and the boundary conditions of the problem, author investigates the solution $w(x, y, \nu + \epsilon)$ expressed in the power series of ϵ, if the deflection $w(x, y, \nu)$ is known.

A numerical example is given, solved by the method of finite differences.

C. Hoschl, Czechoslovakia

4054. Bassali, W. A., and Hanna, N. O. M., Bending of curvilinear and rectilinear polygonal plates symmetrically loaded over a concentric circle, *Proc. Camb. Phil. Soc.* **57**, 1, 166–179, Dec. 1960.

Using complex variable methods and a special class of polynomial mapping functions, many different problems are solved exactly or approximately. Numerical values for the center deflection and maximum moments are listed. By truncating infinite series, approximate mapping functions are obtained for rectilinear polygons. Results for a central concentrated load compare well with known results found by other methods.

R. A. Clark, USA

4055. Bassali, W. A., Some problems in the small deflexions of clamped thin isotropic plates (in English), *ZAMM* **40**, 10/11, 493–507, Oct./Nov. 1960.

Closed form solutions are given for some problems in the bending of clamped plates. The boundaries are such that they can be mapped onto the unit circle by the expression $z = c\,\xi/(1 + \lambda_1 \xi^n + \lambda_2 \xi^{2n})$, $c > 0$. The solutions are obtained by conformal mapping methods. Solutions are given for loads distributed symmetrically over the area of a circle or concentrated forces and couples acting on plates bounded by inverses of an ellipse with respect to internal or boundary points and regular curvilinear polygonal plates bounded by n equal and nearly circular arcs.

E. H. Dill, USA

4056. Brotchie, J. F., Elastic-plastic analysis of transversely loaded plates, *Proc. Amer. Soc. Civ. Engrs.* **86**, EM 5 (*J. Engng. Mech. Div.*), 57–90, Oct. 1960.

In limit design full development of the plastic strength is the object but deformation in the plastic range, particularly in thin plates, may be a more limiting criterion of failure. In this paper the author develops equations giving solutions for small deflections due to axisymmetrical and unsymmetrical yielding along lines in a plate and axisymmetrical yielding over an area. Scope is limited to an ideal elastic-plastic plate with no strain-hardening range, uniformally thin, and composed of a homogeneous isotropic material obeying the Tresca criterion for yield. Many of the results are applicable also to reinforced-concrete plates and the axisymmetrical yielding solutions are suitable for design use and may be extended to nearly symmetrical bending, particularly yielding around cylindrical supports in a continuous plate structure. An appendix develops equations for moment and shear interactions and paper concludes with a useful bibliography.

A. F. W. Langford, Australia

4057. Boal, J. L., and Reissner, E., Three-dimensional theory of elastic plates with transverse inextensibility, *J. Math. Phys.* **39**, 3, 161–181, Oct. 1960.

This interesting paper considers the exact solution of the equations for an elastic plate of arbitrary thickness under the assumption that the material is inextensible in the transverse direction (i.e., that one of the stress-displacement equations is replaced by the condition that the transverse displacment is independent of the thickness coordinate z). In this theory boundary tractions parallel to the plane of the plate must be prescribed both as to distribution in that plane and with z, while the resultant only of transverse shear can be stipulated. The solution is given in terms of three "stress functions," which are coupled through the boundary conditions. A procedure for effectively handling this problem through the use of the principle of minimum potential energy is illustrated in an example. This example, which is solved in detail, pertains to an infinite plate with a circular hole under tractions which in a plate without a hole would correspond to homogeneous transverse bending. Detailed comparisons with the elementary and other plate theories are carried out.

B. A. Boley, USA

4058. von Gunten, H., Plates with unsupported edges (in German), Mitt. Inst. für Baustatik no. 35, 94 pp., 1960.

Small-deflection theory of thin elastic rectangular plates with all edges free is discussed. The load, in the form of pressure varying in one direction symmetrically with respect to the plate's axis of symmetry, is equilibrated by reactions uniformly distributed over a small rectangular area at the center of the plate. Use is made of the known solution for a plate under the same loading conditions with two opposite edges free and the other two simply supported. Reactions distributed along the supported edges are replaced by a system of statically equivalent concentrated forces acting at suitably selected points; these forces are neutralized by means of a series of biharmonic "correction functions" which do not affect the boundary conditions at the free edges. (The method of nodal-points-loading is developed by F. Stüssi in his book "Baustatik" I., Birkhaüser, Basel, 1946.)

A numerical example is given. Two chapters contain valuable hints on simplifying the calculations, and two others deal with the extension of the proposed method to plates on elastic foundation, and plates supported by more than one column.

Reviewer believes that the clear presentation of the proposed method and the simplifications discussed in the last chapters make the book valuable for the practicing engineer.

A. Kornecki, Israel

4059. Dhaliwal, R. S., Bending of isotropic thin square plates by concentrated edge couples and forces, *Appl. Scient. Res.* **(A)9**, 5, 297–318, 1960.

In this paper the complex variable method of Muskhelishvili is applied to the problem formulated in the title. The necessary con-

formal mapping function is found from the Schwarz-Christoffel formula. A general solution is given for plates which can be mapped on a unit circle by polynomial-type mapping functions. Three particular problems are worked out in detail; namely those of approximately square plates subjected, respectively, to two bending couples, to two twisting couples, both applied at the ends of a diagonal, and to four forces applied at the four corners. Numerical results are presented in the form of tables and graphs.

From author's summary by H. C. Reggini, Argentina

4060. Popov, G. Ya., The coupled, integro-differential equations of an infinite flat plate with discretely constant rigidity resting on an elastic semi-space (in Russian), *Izv. Vyssh. Uchebn. Zavednii. Matematika* no. 1, 195–209, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4209.

Author examines problem of a plate consisting of two semi-infinite plates of constant rigidity, in the absence of friction and adhesion between the plate and the semispace, and loaded according to a law $q(x, y) = q_\lambda/(x) \cos \lambda y$ by a concentrated force. The problem is reduced to the solution of coupled, integrodifferential equations; the solution, found formally in closed form, is verified, and its validity proved. A. G. Ishkova

Courtesy Referativnyi Zhurnal, USSR

4061. Galletly, G. D., Optimum design of thin circular plates on an elastic foundation, *Proc. Instn. Mech. Engrs.* **173,** 27, 687–698, 1959.

The problem of a thin circular plate under uniform normal load and supported on an elastic foundation is solved subject to elastic restraints at the edges. Classical plate theory is used, and axial symmetry supposed. Curves are given for determination of the maximum stress, which is shown to be decreased by 25 to 50% over that found in simply supported plates, a result of interest to designers. A brief discussion of the application of the theory to perforated plates and of some design factors concludes the paper. Experimental verification of the theory is not furnished.

W. P. Graebel, USA

4062. Popov, G. Ya., Bending of an unbounded plate supported by an elastic half-space with a modulus of elasticity varying with depth, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 6, 1566–1573, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

The elastic half-space is assumed to have a modulus of elasticity which increases with the depth Z according to the law, $E = E_\nu Z^\nu$, where $0 \leq \nu < 1$. The punch problem for such a half-space was previously considered by Korenev [AMR **12**(1959), Rev. 4886] and Mossakovskii [AMR **12** (1959), Rev. 4324]. Assuming the plate is loaded normally in a special manner, the author derives an integrodifferential equation for the pressure under the plate and obtains integral representations of the pressure and the plate deflection in terms of rapidly convergent integrals. Special attention is given to a plate with a concentrated line load and simple formulas are obtained for the maximum bending moment in the plate and the maximum pressure under the plate.

R. A. Clark, USA

4063. Gondikas, P., and Salvadori, M. G., Wind stresses in domes, *Proc. Amer. Soc. Civ. Engrs.* **86,** EM 5 (*J. Engng. Mech. Div.*), 13–29, Oct. 1960.

The bending stresses due to antisymmetrical wind pressures in a hemispherical dome elastically built into a cylinder are determined by an approximate procedure whereby the relative displacements at dome and cylinder due to wind loading are made compatible by the approximation of using cylindrical theory for the dome as well as for the cylinder. Solution is valid when ratio of dome thickness to radius is less than about 1/50, thus covering most practical cases. R. D. Milne, England

4064. Cicala, P., Membrane stresses in hyperboloid shells of revolution, *Proc. Amer. Soc. Civ. Engrs.* **86,** EM 5 (*J. Engng. Mech. Div.*), 147–159, Oct. 1960.

The computation of membrane stresses and of the corresponding deformations for a hyperboloid shell of revolution under the most general loading is reduced to the evaluation of simple integrals. For the determination of wind effects, diagrams capable of simplifying computations are presented and an approximate formula is suggested.

From author's summary by R. D. Milne, England

4065. Reiss, E. L., A theory for the small rotationally symmetric deformations of cylindrical shells, *Comm. Pure Appl. Math.* **13,** 3, 531–550, Aug. 1960.

Paper seeks to rectify the known inadequacies of the usual approaches to shell theory by deriving, from the "exact" theory, approximate thin and thick shell theories. Stress and displacement components of cylindrical shell, radius R and thickness $2b$, are represented by power series expansions in ϵ, a small parameter which is a positive power of b/R. These series are substituted in elasticity equations, and coefficients of like powers of ϵ equated. Small powers of ϵ yield classical equations; higher powers yield "thick shell" corrections. To satisfy boundary conditions, generalization of boundary-layer expansion technique, due to Friedrichs, is developed. Choosing cylindrical coordinates r, θ, x, shell is considered as three-dimensional body bounded by coaxial cylindrical surfaces, $r = R \pm b$, $x = 0$, L. Homogeneity, isotropy, "first order" theory are assumed. Force system and displacements are rotationally symmetrical. Hence only nonvanishing stresses and displacements are functions of x, r.

Analysis is clear but entirely theoretical. Paper contains neither diagrams nor worked example. Reviewer opines, from practical and research experience, that such purely theoretical work, while intrinsically impeccable, cannot serve the needs of engineers without ensuring that numerical calculations, in a specific example, yield a set of displacement values which are feasible and reasonable, and not merely correct mathematically.

F. A. Gerard, Canada

4066. Raetz, R. V., An experimental investigation of the strength of small-scale conical reducer sections between cylindrical shells under external hydrostatic pressure, David W. Taylor Mod. Basin Rep. 1397, 27 pp., Mar. 1960.

This is the third report in a series [AMR **10**(1957), Rev. 2860 and AMR **13**(1960), Rev. 4575] on the experimental determination of structural behavior of truncated cones under hydrostatic pressures. Six models of various combinations of cones angles, thicknesses of cone and cylinder plating, and of different cone-cylinder reinforcements were tested. The models were of much smaller scale and the shells had longer length-to-thickness and diameter-to-thickness ratios than those of the second series.

The measured elastic strains agreed well with strain calculated by the approximate method suggested by Raetz and Pulos [AMR **13**(1960), Rev. 3329]. Study of the measured and theoretical strains indicated that, for the range investigated, reinforcement at the cone-cylinder junctures should have little effect on the collapse strength of the conical sections. The observed collapse pressures, although affected by initial imperfections, lend support to this contention. T. H. H. Pian, USA

4067. Sanders, J. L., Jr., Nonlinear theories for thin shells, Harvard Univ., Div. Engng. Applied Physics, Cambridge, Mass., TR no. 10, 30 pp., Feb. 1960.

A large deflection theory for thin shells is developed. Transfer shear and normal strains are neglected. Tensor notation is used in the derivation of equations, but the principal results are repro-

duced in the ordinary notation in the appendix. Certain simplifying assumptions such as small strain approximation, small strain and moderately small rotations are considered. Linearized forms of equations coincide with small deflection theories found in the literature. O. Gurel, USA

4068. Visarion, V., and Stanescu, C., Formal reducing of the global state of stress of elastic thin shells to the quasi-invariant state of pure moments (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **11**, 5, 1195–1199, 1960.

Authors use the quasi-invariant concept introduced in a previous paper [*Prikl. Mat. Mekh.* no. 1, 1961] in which the existence of a complex quasi-invariant formulation of the equation of thin shells was proved for the case of anisotropic shells.

Based on this idea, authors emphasize in this new paper the existence of a theoretical model consisting of a shell whose middle surface coincides with the middle surface of the considered shell, having however the imaginary semi-thickness

$$b_x = i\,\frac{b}{\sqrt{3(1-\mu^2)}}$$

This shell acted upon only by quasi-invariant transverse shear forces is equivalent in calculations to the actual shell acted upon by all forces and moments considered in Love's formulation.

In the quasi-invariant formulation the order of the systems of equations and the number of unknowns is reduced by half, the quasi-invariant formulation being equivalent to the classical formulation of shells. A. Petre, Roumania

4069. Zimin, V. I., The application of the method of asymptotic integration to the solution of a particular equation of the theory of shells (in Uzbek.) Russian), *Uz. SSR Fanlar Akad. Akhboroti. Fiz.-Matem. Fanlari Ser.* no. 4, 97–103, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4147.

An ordinary sixth-order differential equation is examined, the integration whereof is performed by applying the problem of small oscillations of thin flat axially-symmetrical shells of rotation. Following I. Ia, Shtaerman [*Izv. Kievok. Politekhn. i S.-Khoz Intov*, **1**, 1, 54–72, 1924], the author attempts the construction of a further equation, capable of integration and little different from the starting equation. He succeeds with a degree of error within h/R (h = thickness of shell, R its characteristic radius). The integral of the approximate equation is written in finite form. The process whereby the results obtained are made specific is not indicated. A. L. Gol'denveizer

Courtesy Referativnyi Zhurnal, USSR

4070. Kassimov, D. M., The analysis of a particular, concrete problem of a prismatic shell (in Azerb.), *Elmi Esserler Azerb. Univ.* no. 11, 49–72, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4151.

Author starts from the theory of prismatic shell of I. N. Vekua [*Trudi Tbilissk. Mat. In-ta* **21**, 191–259, 1955]. Equations are derived (in the zero approximation) for flat, prismatic shells and, in particular, for shells bounded by surfaces (plates of varying thickness). The equations of the zero approximation of shells bounded by planes are solved by the methods developed by N. I. Muskhelishvili ["Some fundamental problems of the mathematical theory of elasticity," 4th ed., Moscow, Izdvo Akad. Nauk SSSR, 1954]. A. L. Gol'denveizer

Courtesy Referativnyi Zhurnal, USSR

4071. Krasiukov, V. P., The calculation of flat shells by the method of finite differences (in Ukrainian), *Nauk Zap. Kievsk. In-ta* **16**, 16, 247–258, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4153.

Author examines flat shells extending over a rectangular planform and loaded by superficial vertical forces. The fundamental system of differential equations for the displacements is written in the form given by A. A. Nazarov [*Prikl. Mat. Mekh.* **13**, no. 5, 1949]. These equations are presented in terms of the finite differences for the intermediate joint of a rectangular lattice. For shells of square planform, results obtained by the method of double, trigonometric series, and the method of finite differences are compared for four different ratios of the camber of the shell to its thickness: 1, 3, 5, 10. P. M. Varvak

Courtesy Referativnyi Zhurnal, USSR

4072. Solianik-Krassa, K. V., The compression and flexure of open, spherical shells (in Russian), *Trudi Leningrad Politekhn. In-ta* no. 192, 71–80, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4160.

An examination (in the light of the general theory of elasticity) of the case of an axially symmetrical spherical dome of constant thickness with a lantern cutout. The shell is regarded as being bounded by external and internal surfaces in the form of concentric spheres, and lateral surfaces consisting of coaxial cones with a common apex at the center of the spheres. It is assumed that the shell is loaded only on the lateral surfaces: by given forces on the lantern cutout, and by reactive forces on the bearing surface. A solution is presented for the compression of the shell by uniform axial forces; the pure bending of the shell by a moment vector perpendicular to the axis; and the bending of the shell by a moment vector perpendicular to the axis; and the bending of the shell by a force perpendicular to the axis. The solution is obtained by applying stress functions. The conditions for the absence of load on the external and internal surfaces are rigorously satisfied. On the lateral surfaces, modified static conditions are applied (the requirement of static equivalence of the superficial forces and the forces and moments satisfying the problem). The geometrical boundary conditions on the lateral surfaces are disregarded.

A. L. Gol'denveizer

Courtesy Referativnyi Zhurnal, USSR

4073. Togonidze, V. R., The problem of calculating reinforced-concrete bunkers in the form of ribbed, cylindrical shells (in Georgian), Sakartvelos Politeknikuri Institute, Shromebi no. 9 (57), 155–167, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4179.

The cylindrical shell in question has a horizontal generating line, is open at the top, is supported on vertical, rigid and flat diaphragms, and has, in the span, stiffening ribs in the form of inverted arches. The analysis of the shell is performed by the method of the elementary beam; the ribs are calculated for bending in their own plane under the action of loads and reactive shearing forces, transmitted by the body of the bunker. Comparison with a bunker of pyramidal form confirms the advantages of a cylindrical bunker. I. K. Snitko

Courtesy Referativnyi Zhurnal, USSR

4074. Kil'chevski, N. A., Integrodifferential and integral equations of equilibrium of thin elastic shells, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 1, 165–178, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The method used in this work is based on application of the work reciprocity theorem for deriving integrodifferential and integral equations of equilibrium of thin elastic shells in terms of displacements. The paper treats shells of arbitrary form of the middle surface. As an introductory problem, the simplest example of a one-dimensional problem, i. e., a circular arch under concentrated load, is given and a Fredholm integral equation of the second kind is obtained. Then the method is applied to the equilibrium of thin elastic shells. A specific example (spherical dome) is also given at the end. The paper concludes with a dis-

cussion on the question of equivalence and uniqueness of the solution. O. Gurel, USA

4075. Bailey, R., and Hicks, R., Localized loads applied to a spherical pressure vessel through a cylindrical insert, *J. Mech. Engng. Sci.* **2**, 4, 302–311, Dec. 1960.

Solution is obtained for stresses and deformations in thin spherical shell subjected to localized bending moment or radial load which is applied through a thin-walled, flexible, cylindrical insert (for example, a duct connection on a reactor pressure vessel). Reissner's equations for shallow spherical shell are used for the vessel, and the Donnell equations for the cylindrical insert. Equilibrium and compatibility conditions are enforced at cylinder-sphere junction. Results are presented in form of four simultaneous equations expressed in terms of Kelvin functions. A numerical example is given for each method of loading, and it is shown how localized stresses are affected by a change in duct thickness.

Authors evidently were unaware of earlier papers by P. P. Bijlaard which treated same problem in similar manner, and presented extensive numerical results in graphical form ["Stresses in a spherical vessel from radial loads acting on a pipe" and "Stresses in a spherical vessel from external moments acting on a pipe," Welding Research Council Bull., Series no. 49, April 1959]. D. O. Brush, USA

4076. Hetenyi, M., and Timms, R. J., Analysis of axially loaded annular shells with applications to welded bellows, *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 741–755, Sept. 1960.

A method is presented for the calculation of stresses and deflections in ring-shaped shells of the type used in the manufacture of bellows. The differential equations of equilibrium and displacement are solved subject to boundary conditions on the inner and outer edge of the ring shell. Authors reason that for typical bellows construction these edge boundary conditions are zero radial displacement and zero rotation at both edges. Using these boundary conditions they obtain simple design formulas for the maximum stress and deflection. These formulas consider the shell to be loaded on the inner edge by a shear force. An experimental study was made for a shell with such boundary conditions and load, and experimental results generally verified the analytical formulas within fifteen per cent error on stress and less error on deflection.

The equations were also solved permitting rotation of the edges, i.e., simple support. These analytical results were also checked experimentally and provided about the same correlation as the fixed-edge case.

The results appear to be most useful for preliminary design of bellows and similar type shells. D. W. Breuer, USA

4077. Gunther, H., On calculations for circular cylindrical tubes under nonsymmetric loads (in German), *Tecbnik* **15**, 2, 76–79, Feb. 1960.

By introducing a stress function ϕ defined in terms of the in-plane stress resultants and integrals of the surface tractions, author reduces Donnell's equations to an eighth-order differential equation in ϕ. General solutions are found to both the inhomogeneous equation (when surface tractions are expressed as a double Fourier series) and the homogeneous equation. In the latter case, a table is provided to aid in computation of stress resultants, moments, and displacements. W. E. Jahsman, USA

4078. Just, E., Stress calculation of circular cylinder (in German), *Maschinenbautechnik* **9**, 3, 148–154, Mar. 1960.

Thick-walled cylindrical shells of finite length are loaded in radial directions under conditions of circular and lateral symmetry. The ends of the shell are free. Stresses and displacements in radial directions are described by Bessel functions, those in axial directions being described by circular functions. The solution is obtained with the aid of Marguerre's stress function, the compatibility conditions and the boundary conditions in the direction of the cylinder axis being satisfied on the average only. In general, the solution is represented by series. The stresses in the direction of the cylinder axis, whose boundary values are correct on the average only, remain small as compared to the other normal stresses. The method of calculation is illustrated by four practical examples.

G. Sonntag, Germany

4079. Filotti, A., Some remarks on the Carpino method of calculating the interior jacket of prestressed feed pipes (in Roumanian), *Hidrotehnica* **4**, 4, 119–125, Apr. 1959.

Paper uses the theory of elasticity for discussing the problem of prestressing the interior jacket of feed pipes under pressure. Admitting V. Carpino's hypotheses, author calculates the efficient pressures exercised by the injected layer on the jacket and rocks after the setting and shrinkage of the mortar. The stresses which occur are also calculated.

The formulas established are discussed for a general case with the aid of three diagrams. The following two conclusions are drawn: the variation of the prestress cannot be considered *a priori* as negligible and the elastic strength coefficient of the rock has a considerable influence on the ratio of the efficient pressure to the injection pressure. M. V. Soare, Roumania

4080. Uflyand, Ya. S., Hartman problem for a circular tube, *Soviet Phys.-Tech. Phys.* **5**, 10, 1194–1196 (Brief Communications), Apr. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30**, 10, 1258–1260, Oct. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4081. Lantsman, M. Kh., Determination of static characteristics of a force-compensation system containing an elastic tube, *Measurement Techniques* no. 6, 421–426, June 1960. (Translation of *Izmeritel'naya Tekhnika* no. 6, 25–28, June 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

4082. Sobolevsky, V. M., The elastic stress condition of an anisotropic, circularly-cylindrical tube in an elastically-anisotropic medium, acted upon by an internal pressure, an axial load, and a radial heat flow (in Russian), *Dokladi Akad. Nauk BSSR* **1**, 3, 83–88, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4115.

The elastic stress condition of a circularly cylindrical tube is analyzed, the cross section of which is bounded by two concentric circles. The tube is cylindrically anisotropic and transversely isotropic in the radial direction, and is acted upon externally by the pressure of an elastic medium with cylindrical anisotropy and transverse radial isotropy, within which the tube is located. Internally, the tube is acted upon by a pressure p, an axial load P, and a radial heat flow $t = t(\rho, \tau)$. Expressions are derived for the distribution of the stresses and displacements, valid for any temperature function varying according to any law applicable to radial, unsteady or steady, heat flow. Yu. G. Burov

Courtesy Referativnyi Zhurnal, USSR

4083. Trukhlov, A. M., The calculation of circular silos of reinforced concrete (in Russian), *Trudi Saratorsk. Avtomob.-Dor. In-ta* **15**, 1, 3–7, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4178.

The elastic analysis is presented of a closed, circular, cylindrical shell with one edge rigidly constrained and one edge hinge-supported, in application to the calculation of circular silos. The load is assumed to be irregularly distributed over the circum-

ference, and the internal pressure acting along the generating line. The solution is obtained by the method of V. Z. Vlassov ["General theory of shells," Moscow-Leningrad, Gostekhteorizdat, 1949] for the case of an orthotropic shell. The stress function is determined for a particular loading case. Expressions are given for the bending moment and the ring (bursting) forces. Recommendations on the selection of the reinforcement are given for the case of a reinforced-concrete shell. A. D. Pospelov

Courtesy Referativnyi Zhurnal, USSR

4084. Goldberg, M. A., Salerno, V. L., and Sadowsky, M. A., Stress distribution in a rotating spherical shell of arbitrary thickness, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 127–131, Mar. 1961.

An exact solution is presented for stresses in an elastic spherical shell rotating about a diameter. Charts of stress distribution are included. Numerical and graphical data are presented for a variety of radius and thickness values. One interesting feature is the indication that the stresses at the pole on the outer surface are sensitive to Poisson's ratio. H. Becker, USA

4085. Matsunaga, S., On the stress distribution of the rotating circular ring type shell (in English), *ZAMM* **40**, 12, 566–568, Dec. 1960.

The membrane equations for a complete toroidal shell rotating about its axis at a constant rate and subject to internal pressure (automobile tire) are solved. Numerical results are presented in a convenient graphical form for several geometric proportions.

E. H. Dill, USA

4086. Vol'mir, A. S., A survey of research on the theory of flexible plates and shells during the years 1941–1957 (in Russian), Rasschet Prostansh. Konstruktsii no. 4; Moscow, Gosstroiizdat, 1958, 451–474; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4143.

A general review of Russian and foreign analytical papers dealing with finite deflections of envelopes (shells) and plates, including 174 titles, and embracing researches on the fundamental equations of the problem, integration methods, the behavior of plates and shells under transverse loading, stability, and the hypercritical behavior of shells. E. I. Grigolyuk

Courtesy Referativnyi Zhurnal, USSR

4087. Pogorelov, A. V., On the elastic deformation of convex shells in the supercritical region, *Soviet Phys.-Doklady* **5**, 4, 895–897, Jan./Feb. 1961. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **133**, 4, 785–787, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4088. Stuiver, W., An approximate solution for the symmetrical end problem of conical shells, *J. Aerospace Sci.* **28**, 1, 71–72 (Readers' Forum), Jan. 1961.

4089. Baumann, W., Contribution to the snap-through problem in circular bimetallic plates (in German), *Öst. Ing.-Arch.* **14**, 3, 161–189, Oct. 1960.

Plate is subjected to two pairs of concentrated forces acting at the ends of two perpendicular diameters so as to introduce anticlastic curvature, and to a uniform temperature distribution. A large-deflection theory for sharply curved shells developed by Marguerre [Proc. 5th Int. Congr. Appl. Mech., 1938] is employed. With the help of the Galerkin method, an approximate solution is obtained and evaluated numerically. A snap-through effect is found to occur. Comparison of the theory with experimental results shows good agreement. A. D. Topping, USA

4090. Cheng, S., Torsion of sandwich panels of trapezoidal, triangular, and rectangular cross sections—supplement, U.S. Dept. Agric., For. Prod. Lab. Rep., 1874-A, 17 pp., Nov. 1960.

4091. Houghton, D. S., and Chan, A. S. L., Pressure cabin analysis, an extension to existing theory, *Aircr. Engng.* **32**, 375, 126–131, May 1960.

Authors claim that, in the stress analysis of pressure cabins, they have examined the effect of the curvature in the longitudinal direction of the symmetrically deformed cylindrical shell. Actually, they have simply added the effect of the longitudinal membrane force due to the pressure, the treatment of which is well-known and is given, for instance, in the standard reference by M. Hetényi, "Beams on elastic foundation," p. 138.

Y.-Y. Yu, USA

4092. Hanuska, A., On the possibility of applying eigenfunctions to the calculation of quadrilateral plates (in German), *Rev. Mécan. Appl.* **5**, 6, 779–784, 1960.

Author discusses the case of a quadrilateral of arbitrary shape proceeding from the solution of the problem constructed with the help of eigenfunctions for two angular plates which correspond to two opposite vertices of the quadrilateral. The two plates intersect upon a straight line which joins these two vertices. On the straight line, author sets continuity conditions for deformations which determine the sequence of the introduced arbitrary parameters. For fulfilling these conditions, approximate methods of calculation may be used, e.g., the collocation method. An example of calculation shows that the method may be applied without considerable difficulties and results in fairly good approximations. The same method may be employed for similar regions in the plane problem of the theory of elasticity.

P. P. Teodorescu, Roumania

4093. Maliyer, A. S., The stress condition of a thick slab, symmetrical about a median phase (in Russian), *Sb. Leningrad In-ta Inzh. Zh.-d. Transp.* no. 156, 57–88, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4116.

A method developed by the author [*Sb. Leningr. In-ta Inzh. Zh.-D. Transp.* no. 4, 180–244, 1952] of selecting the Ciharmonie functions entering into the general solutions of the Lame equations, as presented by B. G. Galerkin [*Doklady Akad. Nauk USSR, Ser. A* no. 14, 353–358, 1930] is applied to the solution of the stress condition of a thick slab of infinite extension when loaded symmetrically in its median plane. General equations are obtained, which must be satisfied by the initial conditions of the problem, and solutions of the problem are found for the cases of loads distributed according to a paraboloid law, linear in the one and parabolic in the other direction, and a law, parabolic in both directions. The solutions provide for the segregation from the infinitely extending slab, of a slab of finite dimensions, circular for the first wading case, and rectangular in the second.

A. S. Alekseev

Courtesy Referativnyi Zhurnal, USSR

4094. Dlugach, M. I., The analysis of the two-dimensional stress condition in panels with cutouts and panels with stiffening ribs (in Russian), Problems of the Calculation of Dwelling Houses and Public Buildings of Prefabricated Elements, Moscow, Gosstroiizdat, 1958, 157–171; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4204.

From a synthesis of the grid method [cf. P. M. Varvax: "Development and application of the grid method to the calculation of plate structures," Pt. 1, Kiev, Izd-vo Akad. Nauk URSR, 1949] and the method of forces, in the theory of statically-indeterminate bar systems, a method is developed for the calculation of panels with cut-outs and panels with stiffening ribs, loaded in the median plane. Schemes of calculation are presented for the grid method, the order of the calculation, and the determination of the displacements. Valuable examples are given of the calculation of panels with cut-outs and panels with ribs, for which stress diagrams are

plotted, for the horizontal and vertical cross sections in the vicinity of the cut-outs and ribs, under the action of vertical loads.

N. K. Snitko

Courtesy Referativnyi Zhurnal, USSR

4095. Brilla, J., Use of affine transformations for solving orthotropic plates (in Slovenien), *Stavebnicky Casopis* **8**, 1, 45–59, 1960.

Author considers thin plane plates with orthotropy of type II for which the rigidities satisfy the condition $D_{xx}D_{yy} = D_{xy}^2$, the equilibrium equation being

$$D_{xx}\frac{\partial^4 w}{\partial x^4} + 2D_{xy}\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{yy}\frac{\partial^4 w}{\partial y^4} = p(x,y)$$

By using the change of variables $x = x'$, $y = y'\sqrt[4]{\frac{D_{yy}}{D_{xx}}}$, author establishes the load expression and the boundary conditions in the transformed region and shows that the equation reduces to that known for the isotropic case. Thus by using the indicated algebraic transformations the theoretical results for the isotropic case may be applied.

Examples are given for semi-infinite and finite rectangular plates supported on two opposite sides.

M. Misicu, Roumania

4096. Solecki, R., General solution for a thin orthotropic rectangular plate (in English), *Bull. Acad. Polonaise Sci.* **8**, 8, 399–409, 1960.

A Fourier-series solution is given for the state of generalized plane stress in an orthotropic rectangular plate.

Y.-Y. Yu, USA

4097. Bhowmick, S., Effect of an isolated force acting at a point near a rigid circular inclusion in a plate (in English), *ZAMM* **40**, 10/11, 511–512, Oct./Nov. 1960.

Known solution by Sen [*ZAMP* **8**, p. 307, (1957)] for deformation of a plate due to prescribed displacements at a circular boundary is applied to resolve the problem defined in the title.

W. S. Hemp, USA

4098. Yamaki, N., Postbuckling behavior of rectangular plates with small initial curvature loaded in edge compression: Part 2, *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 335–342, June 1960.

Paper extends results of *ASME Trans.* **81** E, pp. 407–414, 1959, to cover stress distribution in buckled plates. Numerical results are presented for a square plate and are used to estimate the load for onset of yielding.

W. S. Hemp, USA

4099. Shioya, S., The effect of square and triangular notches with fillets on the transverse flexure of semi-infinite plates (in English), *ZAMM* **39**, 7/8, 300–308, July/Aug. 1959.

Paper presents a theoretical solution for a semi-infinite plate with a single square or triangular notch (with fillets) subjected to a constant bending moment. The analysis is developed on the basis of the theory of thin plates with small deflections and by using the method of complex representation due to Muskhelishvili. A method of perturbation is adopted to determine the parametric coefficients involved in the solution. The convergence of the perturbation series is not proved but is illustrated by several numerical examples. Numerical results show that for a square notch the stress concentration factor ranges from 1.55 to 1.95 and for a triangular notch from 1.68 to 3.05.

G. C. K. Yeh, USA

4100. Prokopov, V. K., The equilibrium of a hollow cylinder loaded by an axially-symmetrical force (in Russian), *Trudi Leningr. Politekhn. In-ta* no. 192, 43–59, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4114.

The solution of this problem uses homogeneous functions of the axially-symmetrical problem, by means of which the author succeeds in satisfying the boundary conditions at the end faces of the cylinder. However, only the condition of the absence of normal stresses is properly fulfilled, while the absence of tangential (shearing) stress remains unsatisfied. It should be noted that the effect of the action of shearing stresses on the end faces fades at the same rate as do the normal stresses, which is not allowed for in the solution.

V. L. Biderman

Courtesy Referativnyi Zhurnal, USSR

4101. Verma, G. R., Note on bending of rectangular plates by concentrated couples on its edge, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, p. 142 (Brief Notes), Mar. 1961.

Buckling

(See also Revs. 4066, 4089, 4098, 4121, 4129, 4233)

4102. Herrmann, G., and Armenakas, A. E., Shear buckling of bars, *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 455–457, Sept. 1960.

A straight rectangular bar under a state of uniform shear stress is found to buckle in the plane of the loading in a manner analogous to a bar under compression. Both equilibrium and energy procedures are used independently to evaluate buckling equations. Additional insight into the buckling mechanism is provided by the analysis of a model consisting of two rigid bars connected by a rotational spring hinge.

The authors point out that the solution has little practical application because of the extreme degree of slenderness required for the critical shear stress to occur in the elastic range of usual materials. Nevertheless, the existence of this type of buckling phenomenon is of interest.

B. G. Johnston, USA

4103. Kirste, L., Consideration of the problem of minimum weight (in German), *Z. Flugwiss.* **8**, 12, 352–358, Dec. 1960.

The investigation deals with the minimum-weight design of "one-dimensional" elements of construction. Interference between slimness in buckling and local failure leads to an optimum for the "spreading" of the cross section. To make the "useful fatigue" as high as possible, it should occur in the plastic range. Efficiency increases with the "loading density" $P/\sigma_0 l^2$, i.e. with the ratio of applied effort to pure compression strength or yield limit and to the square of the length of reference. Amongst simple sections the round tube is best; polygonal tubes are better, the higher the number of faces. Eventually, they must arrive at the same values. With small over-all loading densities, the plain-wall tubes have to be developed into frameworks to increase the local loading densities.

From author's summary by A. Ylinen, Finland

4104. Schinels, M., Stability of a three-hinge rectangular frame subjected to beam loading (in German), *Bautechnik* **37**, 12, 453–462, Dec. 1960.

Article treats the three-hinge frame uniformly loaded along its beam, using second-order theory, and computes the critical load causing antisymmetric and symmetric buckling. It appears that the narrow frame buckles antisymmetrically while the wide frame will collapse symmetrically. The limit for ideally-elastic materials and equal beam and column stiffness is near $l = 1.5\ b$ (where b is the column height and l the beam length). Furthermore a computation was made of the minimum column slenderness ratio representing the limit of elastic buckling for materials having an elastic limit

(steel). It appears that in practical applications one must always expect to exceed the elastic region.

From author's summary by J. Solvey, Australia

4105. Johnson, D. E., Lateral stability of frames by energy method, *Proc. Amer. Soc. Civ. Engrs.* **86,** EM 4 (*J. Engng. Mech. Div.*), 23–41, Aug. 1960.

Title problem is solved through application of conventional Rayleigh method. Bar deflections are approximated by polynomial expansions leading to fairly good level of accuracy per story. Ambitious claims of over-all accuracy of method are extravagant. In first example approximate results are compared with exact solution of two-story building which has been so proportioned as to make lowest two eigenvalues almost equal; insensitivity of Rayleigh fraction to nature of assumed buckling mode is therefore neither surprising nor typical. Second example is equally unconvincing. No reference is made to extensive bibliography in the field. E. F. Masur, USA

4106. Su, H.-L., On stability of two-dimensional structural frameworks, *Proc. Inst. Civ. Engrs.* **16,** 143–156, June 1960.

A method of stability analysis for structural frameworks based on author's "principle of moment amplification" is suggested. It brings to mind the method used for arches that is also based on the convergence of a process of successive approximations.

Reviewer believes that a more extensive knowledge of method of moment amplification, which appears very interesting, would permit a more accurate study of some sentences that do not appear very clear now. A. J. Bignoli, Argentina

4107. Benthem, J. P., and Vooren, J.v.d., The reduction in stiffness after exceeding the buckling load of simply supported flat panels that change in thickness discontinuously (in English), Nat. LuchtLab., Amsterdam TN S. 558, 15 pp. + figures and tables, Mar. 1960.

Paper starts from the results of J. P. Benthem, published in Nat. LuchtLab. T M S.522., TR.S.527 (1958) and TR S.539 (1959).

An infinite repetition of simply supported flat panels under compression in direction of the supports is investigated by means of linear theory of elastic plates. Panels have in each bay (infinite strip) double thickness that changes discontinuously on a straight line parallel to the hinges.

The reduced stiffness, i.e. derivative of mean compressive stress with respect to mean compressive strain, is calculated for loads in small excess of the buckling load on the basis of formerly developed formulas (N.L.L. TR S.539). Numerical examples are given for two cases chosen from different families of buckling modes and the results are introduced in tables and graphs. In both cases the reduction ratios of stiffness are larger than the value 0.5, which corresponds with the panels of constant thickness.

Starting from these ratios, new formulas for the coefficient of reduction in width of panels, as a generalization of well-known formulas of Karman or Marguerre are tried. A comparison in graphs shows a good agreement with the results of experiments (N.L.L. TM S.522). I. Hlavacek, Czechoslovakia

4108. Stein, M., Loads and deformations of buckled rectangular plates, NASA TR R-40, 71 pp., 1959.

Author relates load and deformation for simply supported rectangular plates that are buckled by normal in plane forces on the boundary. The problem is treated for the case when the edges are loaded by applied forces or by restraint in a temperature rise. The nonlinear plate equations are solved by expanding the displacements in a power series in an arbitrary load parameter. Then, by equating to zero the coefficients of each power of the parameter, a set of linear differential equations are obtained. Author states that successive solutions of these equations will converge to the appropriate result. Numerical results are given for compressive loading along two opposite edges of a plate and they compare well with those of previous investigators. Some excellent experimental results are presented that check very well with the analysis.

R. E. Beckett, USA

4109. Chudzikiewicz, A., Stability loss due to the deformation of the cross section (in Polish), *Rozprawy Inz.* **8,** 1, 47–61, 1960.

A particular form of stability loss is studied for a square thin-walled bar. This form consists in a symmetric elongation of one diagonal and in shortening the other. With such a deformation the axis of the bar remains rectilinear. It is assumed that the end sections are provided with diaphragms rigid in their planes.

The problem is solved both in the elastic and plastic range. In the elastic range two independent methods are used: the integration method of the differential equations for plates constituting the walls of the bar, and the approximate energy method. With the method of integrating the differential equation the deflection function of the plates is assumed in the form $w = f(y) \sin a$, x satisfying the boundary conditions at both ends. Substituting this expression in the equation of the plate an ordinary differential equation of the fourth order is obtained for the function $f(y)$. The integral of this equation contains four arbitrary constants. Two of them are zero for symmetry reasons. For the remaining two a homogeneous system of linear equations is obtained by making use of this system equal to zero, an involved transcendental equation is obtained for the determination of the critical force.

Equations interrelating the dimensions of the bar are obtained in three cases, in which the critical force for the buckling form under consideration is less than the Eulerian force of the bar as a whole. In the discussion it is pointed out that the critical force for the buckling form considered is greater than that obtained for symmetric deformation of the walls found by applying the elementary formula for this case, which can be found in the literature. The possibility of existence of bars is also indicated such that symmetric buckling is hampered and the critical force may prove lower than that for any other form of stability loss.

The approximate solution by means of the energy method is given in view of the difficulties of solving the transcendental equation obtained. The familiar Timoshenko method is used, assuming that the walls are deflected according to trigonometric functions. For buckling in the plastic region the Engesser hypothesis and the Bleich hypothesis are used. The equations obtained differ from the corresponding equations for elastic buckling by some of the coefficients only.

W. Szczepiński, Poland

4110. Thielemann, W., Schnell, W., and Fischer, G., Buckling and postbuckling behavior of orthotropic circular cylindrical shells subject to combined axial and internal pressure (in German), Z. *Flugwiss.* **8,** 10/11, 284–293, Oct./Nov. 1960.

Buckling of geometrically perfect shells is studied. Donnell's equations for a slightly buckled cylindrical shell are generalized to take account of orthotropic character. Two fourth-order nonlinear partial differential equations (equilibrium equation and compatibility equation) are obtained for Airy function ϕ and radial deflection w. It is assumed that w may be represented by a trigonometric expression of Kempner containing six undetermined constants. One of these constants is subsequently eliminated by condition of periodicity of circumferential displacement in the circumferential coordinate. After assumed expression for w is introduced, compatibility equation becomes a generalized biharmonic equation in ϕ. Restricting attention to infinitely long shells, so that boundary conditions at ends are avoided, authors

adopt a particular solution for ϕ. Uniqueness of solution is not discussed, but considerations of periodicity apparently influenced choice of particular solution. Principle of stationary potential energy is used to compute undetermined parameters in w. Five complicated rational algebraic equations are obtained for the five parameters. A scheme for solving these equations with an electronic digital computer was devised, and numerous computed curves are presented and discussed. The characteristic snap-through behavior is illustrated. Work is a significant extension of previous results in nonlinear theory of buckling of cylindrical elastic shells. H. L. Langhaar, India

4111. Langhaar, H. L., and Boresi, A. P., Buckling of a cylindrical shell subjected to external pressure (in English), *Ost. Ing.-Arch.* **14**, 3, 189–203, Oct. 1960.

Authors develop an infinitesimal strain theory, using the principle of minimum potential energy, to determine the critical external pressure on a ring-reinforced cylindrical shell loaded on both sides and ends. Quadratic terms in the strain tensor are retained. A digital computer is used to obtain numerical results.

Critical pressures are found as much as 25% less than those given by classical theory without the empirical modifications often used in engineering applications of shell-buckling theory, the difference being largest for short shells. In some instances the theory was found to give critical pressures lower than experimental values reported in the literature. This is attributed to end-fixity effects, since the theory assumes simply supported ends and flexible end plates. A. D. Topping, USA

4112. Naerlovic-Veljkovic, N., Influence of creep on bearing capacity of reinforced concrete columns (in German), *Ost. Ing.-Arch.* **14**, 2, 99–139, June 1960.

Author assumes a stress-strain law for concrete $\epsilon = \frac{\sigma}{E}(1 - \frac{K}{2}\sigma)$ where E and K are material constants, and $\dot{\epsilon} = \frac{\lambda(t)}{E}\sigma$ as the creep law, where $\lambda(t) = A \exp(-Bt)$ with A and B material constants. The complete linear elastico-viscous law of the material is accordingly assumed as

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E}(1 - \frac{K}{2}\sigma) + \frac{\lambda(t)}{E}\sigma$$

She investigates by numerical methods the bearing capacity of pin-ended reinforced-concrete columns of square section with different initial eccentricities. Assuming the axis of the column to be a half-sine-wave, the calculations are carried out for the two different slenderness ratios of 75 and 100, two initial stresses 0.4 and 0.6, of the critical stress, and 3 initial eccentricities of 0.1, 0.2 and 0.3.

From numerical examples she concludes that tension in the concrete, causing plastic yielding of the steel, is responsible for collapse of the column. However, a rigorous solution would require the knowledge of the stress-strain law also at unloading. Important practical conclusion is danger of short life of such columns even when under comparatively small compressive loads. M. Reiner, Israel

4113. Shesterikov, S. A., On a stability criterion for creep, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23**, 6, 1574–1581, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

The problem considered is a pin-ended column under axial load and the approach to buckling is based on the growth of initial deflection $y = a_{oo} \sin \pi x/L$. The author's definition of stability is novel, "The state of a longitudinally compressed bar will be assumed to be stable if for constant load its initial deflection does not increase faster than a linear function of time; or, in other words, the rate of growth of the deflections with time does not increase and the state is assumed to be unstable if the deflections grow with increasing rate." On this basis the author studies the behavior of a column whose material behavior is represented by plastic strain rate = f (plastic strain, stress).

Reviewer believes that the paper is an important contribution to the subject of creep buckling, although the reluctance to define all symbols, which is common to many Russian papers, may annoy some readers. The dot to denote time differentiation appears to be absent in Eq. [7] and occupies a variety of positions in other equations. I. Finnie, USA

Vibrations of Solids

(See also Revs. 3965, 3974, 3976, 3977, 3979, 4150, 4157, 4168, 4214, 4252, 4253, 4560)

Book—4114. Klotter, K., Technology of vibrations: Vol. 2, Vibration with several degrees of freedom [Technische Schwingungslehre: Band 2, Schwinger von mehreren Freiheitsgraden], 2nd ed., Berlin, Springer-Verlag, 1960, xv + 483 pp. DM 58.50.

This book is the second volume of a work of which the first volume was published in 1937, with a recent second edition. The first volume deals with single-degree-of-freedom vibration in a very complete manner. The present second volume, although part of the second edition of the whole work, is a new publication, dealing with systems of two and more degrees of freedom. The subject matter is quite restricted, and the treatment of this material is very complete and elaborate with ample use of and references to the recent literature. It treats the free and forced vibrations with and without damping of systems with a finite number of degrees of freedom in the classical manner. A clear exposition is given of the two principal electrical-mechanical analogies, and matrix methods are introduced in the last chapter. Tuned vibration absorbers occupy 86 pages; the calculation of torsional natural frequencies of undamped systems by many different methods take 70 pages. References to the literature are numerous and valuable. The text is clearly written and the book is a valuable addition to the literature in mechanical vibration. J. P. Den Hartog, USA

4115. Payne, L. E., and Weinberger, H. F., A Faber-Krahn inequality for wedge-like membranes, *J. Math. Phys.* **39**, 3, 182–188, Oct. 1960.

A useful formula for a lower bound on the lowest eigenvalue of a vibrating membrane lying within a wedge-shaped region is established. Explicit results are obtained for a triangle with two sides of arbitrary length and the included angle of arbitrary size (less than π) and for a sector of an annulus.

E. H. Dill, USA

4116. Schaffers, W. J., The vibration of shaft ropes with time-variable length, treated by means of Riemann's method, *ASME Trans.* **83** B (*J. Engng. Industry*), 1, 68–72, Feb. 1961.

Author obtains a solution of the one-dimensional wave equation applied to longitudinal vibrations in ropes in mining hoists when length of rope is variable in time. Author invokes Reimann's method of characteristic curves and uses differential relations describing conditions of discrete masses on the rope (cages and Koepe pulley) instead of values for boundary conditions of deviation of rope y. No application of the solution is given.

From author's summary by K. W. Hillier, England

4117. Jones, R. P. N., and Mahalingam, S., The natural frequencies of free and constrained non-uniform beams, *J. Roy. Aero. Soc.* **64**, 599, 697–699 (Tech. Notes), Nov. 1960.

This technical note applies classical perturbation theory (see for example, p. 154, "Introduction to theoretical physics," J. C.

Slater and N. H. Frank, McGraw-Hill Company, 1933) to determine linear algebraic equations which are then solved by a method of successive approximations to determine the modal frequencies and shapes of nonuniform beams with various end conditions.

P. G. Kirmser, USA

4118. Wen, R. K., Dynamic response of beams traversed by two-axle loads, *Proc. Amer. Soc. Civ. Engrs.* **86,** EM 5 (*J. Engng. Mech. Div.*), 91–111, Oct. 1960.

An approximate elastic analysis of the response of a simply supported beam under the passage of two-axle loads is presented. The two-axle loads remain in contact with the beam and support another load through springs. The beam is assumed to have an initial profile function and is treated by the classical theory. To effect a solution, it is assumed that the dynamic deflections are proportional to the static deflections due to the moving loads and the weight of the beam. Employing Lagrange's equations of motion, approximate series solutions are obtained. Several interesting numerical solutions are presented and discussed.

No direct experimental evidence or limitations regarding the validity of the fundamental assumption above is presented. However, to support the assumption, reference is made to the experimental work by Hillerborg [Inst. Struct. Engng. Bridge Bldg., Royal Inst. Technol., Stockholm, 1951] for a single-axle load.

D. Frederick, USA

4119. Klint, R. V., and Owens, R. S., The effect of root lubrication on the damping of cantilever beams, *ASLE Trans.* **3,** 1, 149–156, Apr. 1960.

Lubricating the root contact surfaces of a simple cantilever beam will increase the vibration damping of the beam. Experimental evidence is given for cantilever beams of cold-rolled steel, stainless steel, and brass, with contact surfaces dry and greased.

Soft-metal platings as alternate lubricants were used as suggested by the work of Bowden and Tabor. They are shown to produce a significant increase in the damping of cantilever beams; and while this increase is not so great as for greased surfaces, their use appears promising and worthy of further study for possible applications where grease cannot be used.

From authors' summary

4120. Zotov, B. E., The determination of the natural frequency of vibration of a double-lattice bar truss in longitudinal compression (in Russian), *Trudi Saratovsk. Avtomob.-Dor. In-ta* **15,** 1, 108–112, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4240.

An approximate solution is proposed for the problem of the vibration of a longitudinally compressed, double lattice truss with any number of panels. The horizontal inertia forces are neglected. Solution of the equation is reduced to an expression for the square of the natural frequencies.

N. L. Kuz'min

Courtesy Referativnyi Zhurnal, USSR

4121. Slobodkin, A. M., Stability of elastic bars with rigid fins in a supersonic flow (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **27,** 77–80, 1960.

A uniform bar moving with large supersonic speed in longitudinal direction has a rigid symmetric fin attached at its rear end. It is assumed that the aerodynamic forces, acting on the fin surfaces during transversal deflections of the bar, are determined by the piston formula. Stability of solutions of the equation for transversal deflections of the bar under assumed boundary conditions is investigated and it is shown that possibility of flutter exists and that an infinite set of bar oscillations becomes unstable. Evaluation formulas for the corresponding critical speeds are deduced.

A. Kuhelj, Yugoslavia

4122. Lin, Y. K., Coupled bending and torsional vibrations of restrained thin-walled beams, *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 739–740 (Brief Notes), Dec. 1960.

4123. Yamamoto, T., Response curves at the critical speeds of subharmonic and "summed and differential harmonic" oscillations, *Bull. JSME* **3,** 12, 397–403, Nov. 1960.

Lateral vibrations of rotating shaft are recorded, and response curves of subharmonic and "summed and differential harmonic" oscillations are experimentally obtained.

Summarizing, the amplitudes of subharmonic oscillation of 1/2 order increase with the magnitudes of eccentricity of rotating body. Shapes of response curves of subharmonic oscillations of forward precession are those of hard spring type, and the response curves of soft spring type are obtained for backward precessional whirling motions. For one critical speed of "summed and differential harmonic" oscillations, the height of peak decreases as eccentricity increases. The other critical speeds have a constant amplitude, even if the magnitude of eccentricity is changed. Shapes of response curves of "summed and differential harmonic" oscillations are either discontinuous response curves with jump phenomena or continuous, as the nonlinear spring characteristics of shaft are changed.

From author's summary

4124. Mitra, A. K., Note on the torsional oscillations of discs of some simple exponential profiles, *Bull. Calcutta Math. Soc.* **52,** 1, 51–53, Mar. 1960.

Some possible symmetrical modes of torsional oscillations of disks with simple exponential profiles have been considered. Assuming that the disks have no rotation, the case of disks with fixed rims has been discussed.

From author's summary

4125. Ivovish, V. A., The nonlinear oscillations of slightly-curved rods (in Russian), *Izv. Vyssb. Uchebn. Zavedenii. Masbinostroiyenie* no. 1, 27–35, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4236.

The natural oscillations of a hinge-supported rod with an initial curvature of $v_0(x)$ are examined for the case of finite displacements of its central axis. The equation of the transverse oscillations of such a rod has the form

$$EI\frac{\partial^4 v}{\partial x^4} - N\frac{\partial^2 (v + v_0)}{\partial x^2} + m\frac{\partial^2 v}{\partial t^2} = 0 \qquad [1]$$

wherein EI is the bending rigidity, m the mass per unit length, N a longitudinal force applied by a spring acting on a mass concentrated at the free end of the rod. It is assumed that in the static state the curved axis of the rod can be described by the equation

$$v_0(x) = q_0 \sin\frac{\pi x}{l} \qquad [2]$$

where l is the distance between supports. Substituting the solution

$$v(x,t) = q(t) \sin\frac{\pi x}{l} \qquad [3]$$

in Eq. [1] we get the following nonlinear equation

$$M(q,p)dq + N(q,p)dp = 0 \qquad [4]$$

where p is the rate of motion of the wind section of the rod; $M(q,p)$ and $N(q,p)$, are polynomials. Eq. [4] is analyzed by means of a phase plane in the coordinates p, q, the topological structure of which enables the character of the motion to be described for different relationships between the parameters of the system. The relationships between the displacements and the time and period of the oscillations are determined by calculating the integral, which in the general case is hyper-elliptical. In particular cases, the matter is reduced to an elliptical integral and, correspondingly, to

elliptical functions, decaying in particular cases to elementary. A numerical example is included. A. N. Obmorshev

Courtesy Referativnyi Zhurnal, USSR

4126. Kaczkowski, Z., The influence of shear and rotary inertia on the frequencies of an anisotropic vibrating plate (in German), *Bull. Acad. Polonaise Sci.* **8**, 7, 343–349, 1960.

The deflection is taken as a sum of two parts, one based on Kirchhoff's hypothesis and the other due to shears. The plate is subjected to forces in its plane and general normal load and rests on Winkler-type elastic foundation (reaction proportional to deflection). Plate stress-strain relations (averages across thickness) used in equilibrium equation yield two simultaneous coupled differential equations. These are reduced to a single sixth-order equation for a function related to the two components through differential operators. A double sine series solution for the orthotropic rectangular plate with zero shear force in its plane and with supported boundary conditions is given. The vanishing of the denominator gives the characteristic equation. A graph shows the effects of various terms. G. A. Nariboli, India

4127. Finagin, B. A., Investigation of the spectrum and shape of the vibrations of the surface of piezoelectric plates, *Soviet Phys.-Tech. Phys.* **5**, 9, 1045–1056, Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad, Nauk SSSR* **30**, 9, 1115–1123, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

The method and results of an experimental investigation of the spectrum of natural frequencies and the shape of the oscillations of the surface of tourmaline and quartz piezoelectric plates are given; the frequency range is from tens of cycles to several megacycles, embracing various types of vibration—flexural, longitudinal, thickness, etc.

The natural frequencies and photographs of the corresponding shapes of the oscillations of the surface of the piezoelectric plates, obtained by a light interference method, are presented.

On the basis of the experimental data obtained, a comparison is carried out of the different types and shapes of oscillations applicable to the practical use of piezoelectric plates, and a number of conclusions are drawn regarding the method of investigation, the shape and amplitudes of the mechanical oscillations of the surface of the piezoelectric plates, and the relation of the shape of the oscillations with the natural frequencies and the types of oscillations. From author's summary

4128. Tyutekin, V. V., Flexural oscillations of a circular elastic plate loaded at the center, *Soviet Phys.-Acoustics* **6**, 3, 389–392, Jan./Mar. 1961. (Translation of *Akust. Zh., SSSR* **6**, 388–391, July/Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

Paper looks into the solution to the problem of the natural frequencies of axially symmetric flexural oscillations of a circular elastic plate (disk) loaded at the center with an arbitrary load impedance. The characteristic equation is derived and its solution given for the case of inertial loading.

From author's summary

4129. Hersch, J., A method for the evaluation of the vibration or the buckling of clamped plates (in French), *C. R. Acad. Sci. Paris* **250**, 24, 3943–3945, June 1960.

4130. Lin, Y. K., Free vibration of continuous skin-stringer panels, *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 669–676, Dec. 1960.

Determination of the frequencies and normal modes of vibrations for typical airplane skin-stringer panels. Boundary conditions are derived for the general case. Then problem is specialized to row of identical rectangular panels either flat or curved. Good agreement is reported between experimental measurements and computed frequencies. O. Bottema, Holland

4131. Kirk, C. L., Vibration characteristics of stiffened plates, *J. Mech. Engng. Sci.* **2**, 3, 242–253, Sept. 1960.

Frequency equations are derived appropriate to free flexural vibrations of clamped and simply supported rectangular plates having stiffeners parallel to one pair of edges. Comparison is made with experimental values. H. Deresiewicz, Italy

4132. Chu, H.-N., On simple thickness vibrations of thin sandwich cylinders, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 145–146 (Brief Notes), Mar. 1961.

4133. Baltrukonis, J. H., Free, transverse vibrations of a solid elastic mass in an infinitely long, rigid, circular-cylindrical tank, *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 663–668, Dec. 1960.

Equations of free vibrations of an elastic solid in polar cylindrical coordinates are solved. Frequency equations corresponding to a rigid circular cylindrical exterior boundary and a solid elastic interior cylindrical mass are established. Coefficients for the natural frequencies are plotted for four natural frequencies for the four lowest cyclic variations in the circumferential coordinate. Poisson's ratio is considered a parameter and author illustrates a rapid means of exploring the roots of the frequency equation based on simplifications of the frequency equations for special values of a Poisson's ratio parameter. Displacement fields are plotted for the first two frequencies of the first three cyclic variations in the circumferential coordinate for a Poisson's ratio of 0.45.

J. E. Duberg, USA

4134. Strunkin, V. A., On the analysis of axial vibrations of discs of axial turbines and compressors (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 11, 100–102, 1958.

4135. Jager, B., Eigenfrequencies of twisted blades (in German), *Ing.-Arch.* **29**, 4, 280–290, Aug. 1960.

Author presents a numerical method for finding the four lowest natural frequencies of flexural vibrations of a twisted nonuniform blade. The coupling between flexure and torsion is not taken into account. Assumed end conditions at blade root are elastically clamped hinge. Assuming the validity of Timoshenko beam theory the method is applicable in the cases where the length-to-width ratio is greater than 4.5. The blade is divided in ten sections of constant parameters.

Starting from the root at any chosen frequency the corresponding values on the blade top can be determined and compared with the end conditions at this point.

Method is especially convenient for a computing machine. Very valuable paper. K. Julis, Czechoslovakia

4136. Jones, R. P. N., A modified energy method for determining natural frequencies, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 146–147 (Brief Notes), Mar. 1961.

4137. Lin, Y. K., and Lee, F. A., Vibrations of thin paraboloidal shells of revolution, *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 743–744 (Brief Notes), Dec. 1960.

4138. Santini, P., Frequencies and modes of free vibrations of multispar shell wings (in Italian), *Aerotecnica* **39**, 5, 250–257, Nov. 1959.

By the application of the equations of equilibrium and the equations of elasticity to a multispar shell-wing structure a set of difference equations is obtained for the flexural oscillations of the structure. By a method of approximation these equations are reduced to a set of partial differential equations which are then solved to obtain the frequencies and modes of the structure. The torsional oscillations are studied by a similar procedure and the

coupling between the flexural and torsional oscillations is examined.

In the opinion of the reviewer this paper is a substantial contribution to the theory of vibrations of complex wing structures.

L. A. Pipes, USA

4139. Glasgow, J. C., and Cronin, J. L., Jr., Chordwise-bending effects on wing-vibration modes, *J. Aerospace Sci.* **27,** 12, 962–963 (Readers' Forum), Dec. 1960.

4140. Jones, J. P., Helicopter vibrations, *J. Roy. Aero. Soc.* **64,** 600, 743–752, Dec. 1960.

4141. Archer, R. R., Small vibrations of thin incomplete circular rings, *Inter. J. Mech. Sci.* **1,** 1, 45–56, Jan. 1960.

The in-plane inextensional vibrations of incomplete thin circular rings are studied. The classical equations of motion which neglect shear and rotary inertia are solved for the modes and frequencies for ring segments with fixed ends. Numerical evaluations are presented for the first three modes of ring with fixed ends subtending angles of 180 to 360 degrees.

A solution is also given for the response of an incomplete ring fixed at one end with a prescribed motion at the other end.

M. V. Barton, USA

4142. Bondar', N. G., Timoshenko, V. V., and Vysochin, B. M., The natural oscillations of rigid bridge arches (in Russian), (Tr.) Dnepropetr. In-ta Inzh. Zh.-D. Transp. no. 27, 63–84, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4237.

Both flat and steep arches are examined. In the first case, the deformation and weight of the pillars of the superstructure are neglected, and the longitudinal forces in the pillars are replaced by a distributed force acting on the crown of the arch. Combined solution by the Fourier method of the equations of the arch and the crown is completed by determining the expressions for the frequency of oscillation, and calculating the first four frequencies for symmetrical shapes. In the second case, the concentrated reactions of the pillars are retained and a method of graphical analysis (elastic loading) is applied; if matrix notation is used, the fundamental frequencies of oscillation can be approximately determined for rise-span ratios of the order of 0.1 to 0.5. The calculated results are correlated with the results of model tests.

I. K. Snitko

Courtesy Referativnyi Zhurnal, USSR

4143. Volterra, E., and Morell, J. D., A note on the lowest natural frequency of elastic arcs, *ASME Trans.* **82** E (*J. Appl. Mech.*) 4, 744–746 (Brief Notes), Dec. 1960.

4144. Ford, G., and Haddow, J. B., Determining machine foundation natural frequencies by analysis, *Engng. J. Montreal* **43,** 12, 76–80, Dec. 1960.

An analytical procedure, based on Rayleigh's principle, is presented for determination of natural frequencies of vibration of a six-degree-of-freedom rigid foundation. Authors actually present analysis for three translational modes of vibration only, stating that rotational modes can be calculated by a similar procedure. Reason given for not presenting complete analysis is that mass moments of inertia for machines are not usually available.

Analyses for vertical natural frequencies are compared, for sand and for clay, with empirical plot of Tschebotarioff. Excellent agreement is indicated up to foundation areas of 100 square feet. Analysis and experimental data indicate that vertical natural frequency decreases with increased bearing pressures over range from 1 to 10^4 square feet foundation area. This conclusion is in agreement with the graphical presentation but is opposite to data presented in tables for which column headings may be reversed.

Reviewer questions assumption of a rigid, six-degree-of-freedom foundation of 10^4 square feet or 100 feet square since foundation of this size would probably have insufficient structural rigidity to behave as a rigid body. This factor may account in part for the progressively larger deviation between analysis and the empirical results as foundation area is increased above 100 square feet.

W. J. Worley, USA

4145. Bufler, H., and Kiessling, F., A new method of calculation for torsional vibration of piston engines considering torsion of the second kind (in German), *Ing.-Arch.* **29,** 6, 373–387, 1960.

Authors consider the principal torsional deformations and the primary induced torsional deformations of a crankshaft, following the method of C. B. Biezeno and R. Grammel ["Engineering dynamics," Vol. 4, part 3, Blackie, London, 1954]. After reviewing the known procedure for determining the natural frequencies of a system with N masses, the authors apply a trignometric method, based on the solution of the recurrence formula connecting the moments in adjacent shafts, to determine the frequencies of a homogeneous engine with or without additional masses; as an example, they consider a four-cylinder engine with a flywheel.

G. B. Warburton, England

4146. Sorensen, A., Jr., A discussion of the vibration characteristics of a simple mechanical connection, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 415–422, Nov. 1960.

Transmission and isolation of vibration is considered. Performance characteristics are discussed in relation to type of excitation and critical frequencies associated with distributed mass of connection. Ideas are illustrated by example.

R. N. Arnold, Scotland

4147. Mead, D. J., The effect of a damping compound on jet-efflux excited vibrations; Part 1: The structural damping due to the compound; Part 2: The reduction of vibration and stress level due to the compound, *Aircr. Engng.* **32,** 373, 64–72, Mar. 1960; **32,** 374, 106–113, Apr. 1960.

Author discusses damping of simple aircraft-type structural elements due to addition of Aquaplas, a proprietary damping compound. Theory and experiment are restricted to single-mode bending vibrations of the structural elements acting as beams. Dissipation due to extensional strains only is considered. It is assumed that the damping properties of Aquaplas can be represented by a complex modulus, found experimentally to be about 500,000 $(1 + 0.41\,i)$ lb/in^2. Experiments indicate values of real and imaginary moduli may vary considerably, and that the real modulus may decrease with increase in strain amplitude. Author shows that covering only the middle 40 per cent of a beam vibrating in its fundamental mode provides optimum damping in that mode. Effects of frequency and temperature are not considered.

In Part 2, author extends previous work to include cylindrical shells and flat panels. Discussion is restricted to single-mode vibrations. Some reduction in stress levels are indicated, but author regards results as not conclusive.

J. C. Burgess, USA

4148. Tanimoto, B., On the displacement-function for the axially symmetrical visco-elastic vibration (in English), *ZAMM* **40,** 4, 189–190, Apr. 1960.

It is shown how to find a potential function (for displacements) for the problem of the title. The constitutive equations of the medium are those of a Voigt solid with four constants λ, μ, λ', μ'. A potential function for axially symmetric thermoelastic problems is also given.

J. L. Brenner, USA

4149. Zaidel'man, R. L., On a method of handling the results of logarithmic decrement measurements, *Indust. Lab.* **25,** 10, 1288–1291, Aug. 1960. (Translation of *Zavod. Lab., SSSR* **25,** 10, 1233–1234, Oct. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Paper presents a somewhat disconnected discussion of the relationship between amplitude, logarithmic decrement, and number of cycles in a free damped vibration. In particular, an expression is obtained for computing the absolute error in the measurement of logarithmic decrement from an oscillogram record in terms of the number of cycles spanned by the record, and the logarithmic decrement itself.

The result is useful but not general. It is restricted to small values of the decrement and, for numerical results, requires that an estimate be made of the accuracy with which amplitude can be measured from the particular oscillogram record. The expansion of the expression for logarithmic decrement [Eq. 8] should contain factorial denominators.

W. J. Moreland, USA

Wave Motion and Impact in Solids

(See also Revs. 4025, 4037, 4116, 4176, 4212, 4446, 4501)

4150. Sternberg, E., On the integration of the equations of motion in the classical theory of elasticity (in English), *Arch. Rational Mech. Anal.* **6,** 1, 34–50, Aug. 1960.

This is a very useful review of the "general" solutions proposed so far for the displacement equations of the classical theory of elasticity. Particular attention is paid to Lamé's expression for any dynamic solution in terms of one scalar and one vector potential: a clear version of Duhem's proof of completeness of this expression is given; the interesting remark is made that the potentials may depend on time even when they are used to represent solutions for the equations of equilibrium. The relations which exist between Lamé's equation and that of Somigliana-Iacovache are established; the expressions due to Papkovich-Neuber, Galerkin (for equilibrium conditions) and Noll (for steady-state oscillations) are also considered. A final section deals with the special forms which the mentioned expressions assume in the cases of plane strain, plane stress or cylindrical symmetry.

G. Capriz, England

4151. Miklowitz, J., Flexural stress waves in an infinite elastic plate due to a suddenly applied concentrated transverse load, *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 681–689, Dec. 1960.

Author obtains solutions, using Laplace transform technique, for transient bending moments and shear force in infinite plate subjected to concentrated step force at one point (the origin) and to rectangular pulse of force applied at this point. Basic equations used take account of shear deformation and plate element rotation out of the plate plane in an approximate manner, as the Timoshenko equation does for beams. Range of validity of solution of approximate equations is investigated by comparison of curves showing time of arrival of waves of various predominant periods, these curves being drawn for both approximate and exact theories.

H. J. Plass, USA

4152. Kalnins, A., and Naghdi, P. M., Propagation of axisymmetric waves in an unlimited elastic shell, *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 690–695, Dec. 1960.

Propagation of sinusoidally varying waves in a shallow spherical shell of infinite extent, arising from oscillating concentrated force at apex, is studied theoretically. Theory takes account of both radial and meridional displacements. By means of Laplace transform technique expressions for displacement in radial direction and for driving point impedance are obtained. Numerical results for radial displacement as function of distance from point of load application, time and frequency of applied force are presented.

H. J. Plass, USA

4153. Serbin, H., Intense stress field produced in a semi-infinite elastic solid by a bomb blast at the surface, *J. Acoust. Soc. Amer.* **32,** 10, 1250–1256, Oct. 1960.

It is assumed that the time which has elapsed after the explosion is sufficiently small for the pressure on the surface of the solid to be given by Taylor's similarity solution [G. I. Taylor, *Proc. Roy. Soc. (A)* **201,** p. 159, 1950; AMR **4**(1951), Rev. 1381]. It is shown that two stress waves are produced in the solid, a "push" wave and following it a "shake" wave. The pressure and the displacement perpendicular to the free surface are propagated without charge. No reference is made to the effect of the temperature rise behind the blast wave which may be not without significance.

K. Stewartson, England

4154. Serbin, H., Propagation of intense shock waves into the earth, *J. Acoust. Soc. Amer.* **32,** 10, 1257–1262, Oct. 1960.

The theory of the previous paper is modified to take account of properties of the solid occurring in a practical problem. It is assumed that the initial high pressure pulse closes up any voids in the solid and that subsequently the compacted medium behaves like an elastic solid. A comparison with some experimental data shows fair agreement.

K. Stewartson, England

4155. Chao, C.-C., Dynamical response of an elastic half-space to tangential surface loadings, *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 559–567, Sept. 1960.

Closed-form solutions are obtained with the aid of Laplace and Hankel transforms for tangential and vertical displacements at the surface and in the interior below the applied force in an isotropic elastic half-space under the action of a suddenly applied concentrated tangential force in the plane boundary.

It is shown that the Betti-Maxwell reciprocity relation is preserved under dynamic conditions.

A. M. Freudenthal, USA

4156. Deresiewicz, H., Effect of boundaries on waves in a thermoelastic solid: Reflexion of plane waves from a plane boundary, *J. Mech. Phys. Solids* **8,** 3, 164–172, Aug. 1960.

The author expands the results of a previous paper. In the previous paper he shows that in a medium of indefinite extent, the field equations of linear thermoelasticity predict the existence of two plane dilatational waves, one predominantly elastic and the other predominantly thermal, and a purely elastic shear wave. The first two types are dissipative and dispersive and the latter is of constant velocity and amplitude.

In the present paper the author investigates the reflection of the three types of waves from a plane, traction-free, thermally radiating surface. He shows that an obliquely incident dilatational wave of either kind generates reflected waves of all three kinds, each of which is dissipative and dispersive. In the case of an obliquely incident shear wave there results reflected waves of all three kinds. However, the reflected shear wave remains elastic whereas the reflected elastic and thermal dilatational waves have amplitude attenuation only when they displace normal to the boundary.

R. L. Bisplinghoff, USA

4157. Konenkov, Yu. K., Plate waves and flexural oscillations of a plate, *Soviet Phys.-Acoustics* **6,** 1, 52–59, July/Sept. 1960. (Translation of *Akust. Zh., USSR* **6,** 1, 57–64, Jan./Mar. 1960 by Amer. Inst. Phys., New York, N. Y.)

Homogeneous flexural wave propagation along elastic plate strip of finite length and finite width. Solutions are presented for edges free, clamped, and hinged.

Numerical results are presented for some eigenvalues. Results are experimentally verified for free-edge solution.

R. A. Eubanks, USA

4158. Spencer, A. J. M., The dynamic plane deformation of an ideal plastic-rigid solid, *J. Mech. Phys. Solids* **8**, 4, 262–279, Nov. 1960.

This paper is concerned with the motion under strain conditions of an ideal plastic-rigid solid, when the inertial forces are not negligible. It is shown that the equations for such a motion can be expressed in a particularly simple form when referred to a system of moving curvilinear coordinates which is a generalization of the system defined by the slip-line field in quasi-static plane plasticity. A perturbation method of solving the equations is described; in this, approximate solutions to dynamic problems are obtained as perturbations of a known (usually quasi-static) solution. The method is illustrated by application to problems of indentation of a plastic-rigid half-space by stationary and moving loads. From author's summary by E. J. Scott, USA

4159. Sagomonyan, A. Ya., Penetration of pointed axially-symmetric bodies, *Soviet Phys.-Doklady* **5**, 5, 989–992, Mar./Apr. 1961. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **134**, 6, 1320–1323, Oct. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4160. Sankaranarayanan, R., A note on the impact pressure loading of a rigid plastic spherical shell, *J. Aerospace Sci.* **28**, 1, 77–78 (Readers' Forum), Jan. 1961.

4161. Fowles, G. R., Attenuation of the shock wave produced in a solid by a flying plate, *J. Appl. Phys.* **31**, 4, 655–661, Apr. 1960.

Article deals with the attenuation of a plane shock wave produced in a solid by a flying plate of the same material. The attenuation arises from a rarefaction wave which travels at elastic velocity–some 20% higher than the original plastic (hydrodynamic) shock wave. A numerical example is considered and an experiment suggested. Reviewer questions whether spalling associated with the generation of the rarefaction wave may not disturb the author's predictions. F. E. Reed, USA

4162. Berger, J., and Joigneau, Suzanne, On the linear relation between the particle velocity and the speed of shock-wave propagation in metals (in French), *C. R. Acad. Sci. Paris* **249**, 23, 2506–2508, Dec. 1959.

Soil Mechanics: Fundamental

(See also Revs. 4049, 4062, 4519)

4163. Bishop, A. W., and Morgenstern, N., Stability coefficients for earth slopes, *Géotechnique, Lond.* **10**, 4, 129–150 + charts, Dec. 1960.

Theory is presented deriving the factor of safety in terms of slope, angle of friction, ratio of cohesion to height times density, depth to firm layer and pore pressure divided by height times density (pore pressure ratio, r, assumed constant throughout slope). Minimum safety factors were obtained by a digital computer and expressed as $m - rn$. Values of m and n are tabulated and plotted for slopes from 2:1 to 5:1, angles of friction from 10 to 40 degrees, ratio of cohesion to depth times density from 0 to 0.05 and depths to firm layer from 1 to 1.5 times the height. The valuation of the pore pressure ratio is discussed theoretically and by comparison with measured pore pressure distribution.

E. S. Barber, USA

4164. Kezdi, A., Contributions to the bearing capacity of piles (in English), *Acta Techn. Acad. Sci. Hungaricae, Budapest* **29**, 3/4, 275–312, 1960.

Direct shear tests were made with sand against various solids, and a formula was derived for the constant shape of the shear stress-displacement relationship. This, with lateral pressures related to displacement, was used to calculate the distribution of stress along piles with and without point resistance. Calculations are compared with published field measurements and show the importance of displacement considerations.

Model concrete piles 10 × 10 cm from 0.4 to 2 m long were driven into sand and test-loaded. The bearing capacity was found to be a function of the length squared in accordance with theory.

E. S. Barber, USA

4165. Mohan, D., Jain, G. S., and Kumar, V., Bearing capacity of piles by load tests, *J. Inst. Engrs., India* **41**, 4 (Part 1), 163–170, Dec. 1960.

Load tests were carried out on two 40-cm (16 in.) diameter, 23 m (75 ft) long cast *in situ* concrete piles. The load was applied by hydraulic jacks and the reaction was obtained from R.S. joists strapped down to anchor piles. Cyclic loading was used and elastic rebound of the pile top was obtained at every stage, which enabled separation of skin friction and point bearing. The point bearing capacity was also obtained from penetrometer tests and the expression developed by van der Veen. The load-carrying capacity of the pile was estimated from loading tests by different building codes, and it was observed that the minimum value was 47 tons as against 40 tons which had been specified by the firm.

From authors' summary

4166. Berry, D. S., An elastic treatment of ground movement due to mining: Part 1, Isotropic ground, *J. Mech. Phys. Solids* **8**, 4, 280–292, Nov. 1960.

So long as the excavation is comparatively deep the seam thickness may be considered infinitesimal and the convergence of opposing points in roof and floor may be treated as a discontinuity in displacement at a single point. A result in plane elasticity can then be used to find the solution for a partially closed excavation of infinite length in an infinite medium, and from this solution are derived those for an unclosed and a completely closed panel. The corresponding results for a semi-infinite medium are found only approximately for the unclosed and partially closed panels, but exactly for complete closure. In the last case the expression for the displacement at the plane boundary is independent of the elastic constants and forms an unequivocal limit with which actual subsidence profiles may be compared. In British coalfields the magnitude of maximum subsidence is almost always greater than that given by the bounding expression, and the discrepancy cannot be explained on grounds of minor departures from the basic assumptions. It follows that these assumptions must be drastically modified before agreement may be expected.

From author's summary

4167. Mogilevskaya, S. E., The nature of the resistance of loess soils (in Russian), Dokladi 16th-i Nauchn. Konferentsee Prof.-Prepodavat. Sostava Leningrad Inzh.-Stroit. In-ta, Leningrad, 1958, 164–169; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4314.

Experimental investigations are described to elucidate the nature of the resistance of loess soils in the Tashkent Region. The purpose of the investigation was to elucidate the individual components of the total shearing (slip) resistance (forces of internal friction, cohesion, and structural adhesion), the conditions of

plastic deformation, the influence of the degree of humidity and disturbance of structural bonds, on the change in strength (resistance) of loess soils; and, finally, their thixotropic properties. The investigation was further extended to the leaching of loess soils, indicating the absence of any influence on their strength, of weakening of the cementing bonds by the removal of salts. Prolonged tests were made to study the phenomena of slip and creep, indicating the fading of slip strains in time, associated with changes in the moisture content of the soil and thixotropic phenomena. The results of the investigations are summarized in a series of short monographs.

Z. V. Maslova-Pil'gunova
Courtesy Referativnyi Zhurnal, USSR

4168. Balakrishna Rao, H. A., and Nagaraj, C. N., A new method for predicting the natural frequency of foundation-soil systems, *Struct. Engr.* **38,** 10, 310–316, Oct. 1960.

Authors present a method for computing the resonant frequency of a (machine) foundation-soil system. The method assumes that the resonant vibration of a machine mounted on soil can be adequately characterized by a linear, undamped, single-degree-of-freedom mathematical model. The stiffness value assigned to the model is obtained by a method attributed to Pauw ["A dynamic analogy for foundation-soil systems," ASTM Special Publication no. 156, pp. 90–112 (1953)]. The effective mass considered to be acting in the foundation-soil system is comprised of contributions from the machinery and its foundation, a mass equivalent to the dynamic force acting on the system and the amount of soil considered to be participating in the motion of the system.

Earlier work (e.g., Eastwood [AMR **6**(1953), Rev. 3326] and [AMR **7**(1954), Rev. 405]) has shown that the resonant frequency is a function of a certain mass of the supporting soil (ranging from four to ten times the vibratory load), the shape and dimensions of the loaded area, the magnitude of the machinery mass, the physical characteristics and moisture content of the soil. Authors have attempted to refine this earlier work, particularly, by assuming that the amount of soil participating in the vibration is contained within a so-called intensity pressure bulb defined by authors as "... imaginary boundaries indicating the mass of soil participating in the vibrations...." Authors claim that a separate investigation not disclosed in paper has shown that the appropriate intensity pressure bulb corresponds to that for which the pressure at the boundary equals ρ lb per sq ft, where ρ is the magnitude of the density of the soil in lb per cu ft.

Authors use earlier published results to test adequacy of their method. Using empirical shape factors, average values of various physical parameters, numbers with varying precision and the single-degree-of-freedom assumption, authors display table in which 67 pairs of empirical and computed results are given to four significant figures. The agreement between the experimental results of others and authors' computed results averages less than 2% error, with some where there is no error. Considering the nature of the factors included by authors and the vagaries associated with most dynamic experiments, reviewer is amazed at agreement.

D. Muster, USA

4169. Alekseenko, V. D., Grigoryan, S. S., Novgorodov, A. F., and Rykov, G. V., Some experimental investigations on the dynamics of soft soils, *Soviet Phys.-Doklady* **5,** 4, 727–730, Jan./Feb. 1961. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **133,** 6, 1311–1314, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

Book—4170. Fil'chakov, P. F., Theory of seepage flow under hydraulic structures [Teoriya fil'tratsii pod gidrotekhnicheskimi sooruzheniyami], Vol. 1, Kiev, Izdatel'stvo Akademii Nauk Ukrainskoi SSR, 1959, 308 pp. 12 r.

The reviewed work represents one of several books published in the Soviet Union in the past decade on the theory of seepage flow by followers of the school of Parlovskii and those by the school of Leibenson, both of whom were members of the Academy of Science.

The author treats once more, in Chapter 1, the theory of functions of the complex variable, which is auxiliary to the theory of seepage flow. In Chapter 2 he discusses conformal mapping and its application to boundary conditions typical in the theory of pressure seepage flow. When the theory of conformal transform is explained, the author applies it first to the solution of the most simple cases of seepage flow, and then treats the case of a flat horizontal dam foundation. After that more complicated cases are treated: one sheet pile row under a dam, an underground surface broken at a right angle, which is the case of an irrigation control work, a dam with two sheet pile rows for an infinite and for limited depth of pervious stratum. All these cases were presented and solved in Parlovskii's monograph of 1922 on the theory of seepage flow. They can also be found in the collection of his works published in the fifties. Volume I of the reviewed book contains only two chapters. Volume II will contain three more.

B. S. Browzin, USA

4171. Rethati, L., Capillary properties of soils (in English), *Acta Techn. Acad. Sci. Hungaricae, Budapest* **29,** 1/2, 153–182, 1960.

Sand fractions, two graded sands and loess were allowed to absorb water from the base of a column 6 cm high. Degree of saturation was independent of density but decreased from 1 for uniform material to 0.85 for graded material. Previous wetting reduced the degree of saturation. Above the zone of constant degree of saturation, the distribution of water content is represented by a formula based on continuous readings of the capillary rise and weight of a column of soil. The rate of rise indicated a decreasing permeability with decreasing saturation. Results are compared with published data and hypotheses. Examples of application are presented.

E. S. Barber, USA

4172. Sokolov, A. G., Problems of slip in water-saturated, clay soils (in Russian), *Nauchno-Tekhn. Inform. Biul. Leningrad. Politekhn. In-ta* no. 1/2, 147–155, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4301.

Experimental results are presented of the influence of height of the sample on the deformation capacity of three different soils (heavy sandy loam, diatomaceous clay, and bentonite clay); made in standard, compression-test instruments with a diameter of the earth ring of 70 mm, height 20 mm and 70 mm. It is found that the influence of the height of the sample on the time rate of deformation of the soil depends on the composition and properties of the particular soil. Specifically, in the case of heavy, sandy loam and diatomaceous clay, in which resistance to the expulsion of free water is low, the influence of sample height is least, and is greatest in the case of bentonite clays.

S. R. Meschyin
Courtesy Referativnyi Zhurnal, USSR

4173. Moiseenko, V. M., Investigation of the hydrostatic counterpressure in concretes and soils which have a monolithic structure (in Russian), *Trudi Omskogo S.-Kh. In-ta* **26,** 79–102, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10403.

Concrete (or soil) can be looked upon as a schematic presentation of a porous body with certain conditions attached. For a scheme of this type the magnitudes are found of the principal normal and tangential stresses in the frame of the porous body, account having been taken of the lateral compression by hydrostatic pressure of the particles of concrete (or soil). An investigation is made in a similar manner for a monolithic (nonporous) body. On the basis of the analytical premises which had been worked out, an experimental investigation is carried out on samples

of different concretes, for which, finally, numerical values are found for "the coefficient of counterpressure" α. The two main variants adopted for the plan of research were: (1) experiments where the coefficient of counterpressure is established without destruction of the concrete samples (due to deformation of the concrete sample compressed by hydrostatic pressure); (2) experiments when the value for α is found by destruction of the concrete samples (due to destructive forces acting on the concrete sample because of hydrostatic pressure).

R. R. Chugaev

Courtesy Referativnyi Zhurnal, USSR

4174. Egri, G., and Rethati, L., Conclusions drawn from settlement measurements (in Hungarian), *Mélyépítéstudományi Szemle* **10**, 6, 257–263, June 1960.

Authors conclude from 134 measurements that the bedding coefficient value decreases slightly with an increase of the base surface: The larger the surface and foundation depth, the greater the error in the settlement calculation.

From authors' summary

4175. Kruglov, I. N., Zobachev, N. M., Galitski, V. G., and Rozental, A. I., Automatic equipment for testing soils under static loading (in Russian), *Sb. Nauk-i. In-ta Osnovanii i Podzemn. Sooruzh. Akad. Str-va i Arkhitekt. SSSR* no. 33, 84–99, 1958; *Ref. Zh. Mekh.* no. 34, 1959, Rev. 4316.

A description is given of two automatic, hydraulic equipments devised by the authors for testing soils with the object of determining the compressibility modulus, influence of settling properties, and characteristic behavior of soils under load. One of the test rigs is intended for use supported against the sides of a trial pit; the other, for mounting on an anchoring screw pile. The test equipments are constructed on the principle of maintaining a constant pressure on the soil. The construction of the test rigs is described in detail, as well as the methods of testing soils by these means.

S. I. Galkin

Courtesy Referativnyi Zhurnal, USSR

Soil Mechanics: Applied

(See also Revs. 4163, 4164, 4169, 4171, 4174, 4266)

4176. Vasil'eva, E. F., Prognosis of scour of the storage basin shores (in Russian), *Gidrotekh. Stroit.* **28**, 4, 28–32, Apr. 1959.

On the basis of the author's own observations on three lakes of small depth and on one storage basin, as well as of similar observations and laboratory tests of other authors cited in the paper, instructions are given to facilitate the prognosis of the scour of the basin shores due to the action of waves. While the lower and the upper elements of the profile develop in the very beginning and can be treated as independent of time, the inclination and the underwater depth of the slope in the vicinity of the wave level are parabolic functions of the total wave energy in the given period. Similarly the quantity of the material carried away increases with time, being linearly proportional to the initial slope inclination and to the maximum wave height. The constants of proportionality and the power exponents depend in both cases on the soil character.

The ratio between the material carried away and that deposited at the toe depends on the coarseness of the soil, on the dimensions of the waves and the speed of their flow away from the banks, on the degree of the initial saturation of soil and on the initial bank inclination. In predicting this ratio, the observations corresponding to similar local conditions have to be taken into account.

L. Suklje, Yugoslavia

4177. Turzynski, L. D., Groups of piles under mono-planar forces, *Struct. Engr.* **38**, 9, 286–293, Sept. 1960.

4178. Samoilov, V. P., The forces arising in the penetration of soil by shield edges or force-driven pipes (in Russian), *Vodosnabzhenije i San. Tekhnika* no. 10, 19–26, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4309.

The inconsistency of a number of current views on this subject is pointed out. The force required for penetration into the soil is recommended to be calculated by the formula

$$P_{\text{II}} = P_K + P + P_{B.\text{II}},$$

where P_K is the force of penetration into the soil, of the blunted, cutting edge; P the force required to bring the soil within the cutting part in a plastic state; $P_{B.\text{II}}$ the force required to overcome the friction of the soil along the internal surface of the head (cutting ring) of the shield. The force $P_{B.\text{II}}$ may be disregarded if the soil is continuously removed at a distance from the entry cross section (in the direction of the excavated tunnel), not exceeding 0.15D of the cutting edge. In a first approximation, the force P can be determined by solving the axially-symmetrical problem of the penetration of the cutting edge into an imponderable, friable medium. Applying the relationship $P'_{o(oc)} = P'_o(\text{II}\pi)$, k, between the magnitude of the unit force of penetration into an imponderable medium of two-dimensional ($P'_o(\text{II}\pi)$) and circular ($P'_{o(oc)}$) cutting edges (correspondingly, in conditions of the symmetrical and axially-symmetrical problem), the author reduces the determination of the force P to solution of the corresponding, plane-symmetrical problem. For the latter, the method of graphical analysis of S. S. Golushkevich is used. The force P_K of penetration of the cutting edge is determined by the expression for the two-dimensional problem of the stability of a foundation:

$$P_K = \text{II}\delta_K \left[(\sigma_H + k \operatorname{ctg} \rho) \frac{1 + \sin \rho}{1 - \sin \rho} \exp \pi \operatorname{tg} \rho - k \operatorname{ctg} \rho \right]$$

where $\text{II} = \tau D$. The author illustrates the described method of calculating the total force P_{II} by a concrete example.

I. Govyadinov

Courtesy Referativnyi Zhurnal, USSR

4179. Berezantsev, V. G., Yaroshenko, V. A., Prokopovich, A. G., Razorenov, I. F., and Sidorov, N. N., An investigation into the stability of sand foundations (in Russian), *Trudi Vses. Nauk-i. In-ta Transp. Str-va* no. 28, 140 pp., 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4289.

Results are presented of experimental and theoretical investigations on the nature of the deformations and the conditions for the attainment of the limiting state in the case of sand foundations according to their strength, for different relationships between the depth of the foundation and the density of the sand. Analysis of the experimental data has enabled two essential phases in the deformation of sand foundations to be isolated. The first (initial) phase begins with the instant of application of the load, and ends with formation of a compacted core. The foundation undergoes a process of compression of the soil and formation of local slip planes (discontinuities). A compacted core is formed under the load.

The second phase extends from the instant of formation of this compacted core. The foundation experiences a violent development of slip in large bodies of soil along continuous slip planes. This phase is accompanied by considerable and rapidly growing settling phenomena and lateral displacements of the foundation, presenting danger to the normal existence of the structure. The limiting strength condition of a sand foundation is reached, therefore, at the transition from the first deformation stage to the second. Depending on the density of the sand and the relative sinking of the foundation, four characteristic forms of the limiting state can be distinguished: in sands of high and medium density, when the (building) foundation is erected on the surface or only inconsider-

ably sunk ($0 \leqslant b_e/b \leqslant 0.5$; Case 1a), as also at slightly greater depths ($0.5 \leqslant b_e/b \leqslant 1.0 \div 1.5$; Case 1b), the soil adjoining the foundation is bulged along continuous slip planes; in loose sands, whatever the foundation depth, and in dense and medium-density soils in the case of deep foundations, there is an interaction between the regions of slip and the regions of compaction in the soil, located above ($1 \div 1.5 \leqslant b_e/b \leqslant 3 \div 4$; Case 2a) or below ($b_e/b > 3 \div 4$; Case 2b) the sole of the foundation. In the authors' opinion, the improvement in the method of strength analysis of such foundations should, correspondingly, consist in determining the character of the possible limiting strength condition for sand of a particular density, and a particular depth of foundation. Determination of the resistance of the soil foundation then proceeds by the usual methods of analysis for a limiting state. Analysis of foundations in this manner, according to particular deformation characteristics and conditions for the attainment of a limiting state has, in the cases enumerated, shown good agreement with the experimental results, and disclosed high reserves in the carrying capacity of foundation soils.

A. I. Govyadinov

Courtesy Referativnyi Zhurnal, USSR

4180. Glushkov, A. T., An experimental determination of the contact pressures at the interface of a driven element and an elastically-yielding foundation (in Russian), Dokladi Mezhvuz. Konferentsii po Ispytaniam Sooruzh., Leningrad, 1958, 137–146; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4290.

Experimental results are presented on the driving of rigid and flexible elements into a sand foundation, made in a three-dimensional tray. The contact pressures were measured by membrane boxes of the author's own construction. The investigations extended to the influence of the moisture content of the sand on the nature of the pressure distribution under foundations of varying form and rigidity. According to the author's experimental results, the character of the pressure distribution depends on the cohesion of the foundation, and not on the mean value of the pressure on the foundation, or the extent of the load-transmitting area. Tests with an impressed beam in conditions of symmetrical loading have shown an irregular distribution of the pressure in width: according to a parabolic law for dry sand, and anticlinal (saddle-shaped) in moist sand. For conditions of asymmetrical loading, the critical values have been determined for the eccentricities of the points of application of the concentrated force. It is demonstrated that in analytical calculations it is necessary to take account of the development of regions of plastic deformation under the ends of the beam. A comparison is made between the experimental results and analytical calculations following the theory of an elastic semi-space, using a special, foundation model. The author concludes the suitability of using a composite model for the calculation of foundations on real soils.

L. P. Vinokurov

Courtesy Referativnyi Zhurnal, USSR

4181. Trifonov-Yakovlev, D. A., The compacting of sandy soils by pulsatory action (in Russian), *Trudi Nauk-i. Sektora Mosk., Fil. In-ta "Orgenergostroi"* no. 1, 72–85, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4310.

A discussion of the results obtained by the deep compacting of sands by the action of oscillations induced by pulsations of compressed air. The fundamental propositions for the theoretical analysis of the suggested method are presented. The principle of the experimental arrangement is described, and its trials in the laboratory of the NIS Orgenergostroi. In these experiments, sand was used, containing up to 75–80% fractions below 0.25mm size. The initial, relative density of the sand was 0.1 – 0.2D, corresponding to a specific gravity of $\gamma_{ck} \approx 1.45\ t/m^3$. By the action of a pneumatic, pulsating arrangement, the relative density of the sand was increased over a radius of 2.5m, to a value of $\sim 0.67D$ ($\gamma_{ck} = 1.70\ t/m^3$). A more efficient arrangement for compacting sand is suggested.

Z. V. Maslova-Pil'gunova

Courtesy Referativnyi Zhurnal, USSR

4182. Berry, D. S., and Sales, T. W., An elastic treatment of ground movement due to mining: Part 2, Transversely isotropic ground, *J. Mech. Phys. Solids* **9**, 1, 52–62, Feb. 1961.

The convergence of roof and floor in an infinitely long excavation is treated as a displacement discontinuity in a transversely isotropic medium. The non-closure, partial and complete closure solutions are found for the infinite medium, and the corresponding results for the semi-infinite medium are derived (the complete closure solution exactly, but the other two only approximately). For a deep enough excavation, the shape of the subsidence curve depends upon only two parameters, functions of the four elastic constants involved. A method of deducing these parameters from a graph of measured subsidence is given, and is illustrated by an example. Good correspondence between measured and theoretical curves is found.

The theory is applicable to the problem of closure of a slit in a large plate.

From authors' summary

Processing of Metals and Other Materials

(See also Revs. 4029, 4044, 4548)

Book—4183. Black, P. H., Theory of metal cutting, New York, McGraw-Hill Book Co., Inc., 1961, xi + 204 pp. $7.50.

This textbook is a careful, but elementary analysis of metal cutting mechanism. After very fundamental description of atomic structure and strength of metals, author surveys basic metal cutting mechanics for orthogonal cutting, according to theories of Ernst, Merchant and Shaw, and explains role of friction, lubrication and wear. Following practical chapters deal rather briefly with cutting fluids, cutting tools, tool wear and tool life, and with grinding. Author contributes original and very interesting considerations on three-dimensional cutting, which are helpful in understanding metal removing action in various machining operations. Final chapters describe unconventional machining processes, and survey machining economics.

The textbook is of encyclopedic type and can certainly guide some engineering students to understand metal removal mechanism. For senior tool engineering students or machining research men, it is not complete enough. It does not seem to meet machine shop needs because of an insufficient number of practical recommendations. Specific information on machinability properties of construction materials, and on tool geometries, properties and grades of cemented carbides is lacking. Power consumption for machining is ignored. There are 120 reference listings, covering a large portion of American development, but nearly ignoring foreign development of the art in the past 30 years.

A. Niedzwiedzki, USA

4184. Albrecht, P., New developments in the theory of the metal-cutting process: Part 1, The ploughing process in metal cutting, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 348–358, Nov. 1960.

The accepted theory of metal cutting is claimed to be incomplete. Author suggests the presence of another mechanism, ploughing, in addition to failure by shear as a result of plastic deformation along shear plane. The ploughing phenomenon occurs because the cutting edge is rounded off instead of being perfectly sharp as usually assumed. The small rounding exists not because of any design but because "tiny particles of tool material will

of different concretes, for which, finally, numerical values are found for "the coefficient of counterpressure" α. The two main variants adopted for the plan of research were: (1) experiments where the coefficient of counterpressure is established without destruction of the concrete samples (due to deformation of the concrete sample compressed by hydrostatic pressure); (2) experiments when the value for α is found by destruction of the concrete samples (due to destructive forces acting on the concrete sample because of hydrostatic pressure). R. R. Chugaev

Courtesy Referativnyi Zhurnal, USSR

4174. Egri, G., and Rethati, L., Conclusions drawn from settlement measurements (in Hungarian), *Mélyépítéstudományi Szemle* **10**, 6, 257–263, June 1960.

Authors conclude from 134 measurements that the bedding coefficient value decreases slightly with an increase of the base surface: The larger the surface and foundation depth, the greater the error in the settlement calculation.

From authors' summary

4175. Kruglov, I. N., Zobachev, N. M., Galitski, V. G., and Rozental, A. I., Automatic equipment for testing soils under static loading (in Russian), *Sb. Nauk-i. In-ta Osnovanii i Podzemn. Sovruzh. Akad. Str-va i Arkhitekt. SSSR* no. 33, 84–99, 1958; *Ref. Zh. Mekh.* no. 34, 1959, Rev. 4316.

A description is given of two automatic, hydraulic equipments devised by the authors for testing soils with the object of determining the compressibility modulus, influence of settling properties, and characteristic behavior of soils under load. One of the test rigs is intended for use supported against the sides of a trial pit; the other, for mounting on an anchoring screw pile. The test equipments are constructed on the principle of maintaining a constant pressure on the soil. The construction of the test rigs is described in detail, as well as the methods of testing soils by these means. S. I. Galkin

Courtesy Referativnyi Zhurnal, USSR

Soil Mechanics: Applied

(See also Revs. 4163, 4164, 4169, 4171, 4174, 4266)

4176. Vasil'eva, E. F., Prognosis of scour of the storage basin shores (in Russian), *Gidrotekh. Stroit.* **28**, 4, 28–32, Apr. 1959.

On the basis of the author's own observations on three lakes of small depth and on one storage basin, as well as of similar observations and laboratory tests of other authors cited in the paper, instructions are given to facilitate the prognosis of the scour of the basin shores due to the action of waves. While the lower and the upper elements of the profile develop in the very beginning and can be treated as independent of time, the inclination and the underwater depth of the slope in the vicinity of the wave level are parabolic functions of the total wave energy in the given period. Similarly the quantity of the material carried away increases with time, being linearly proportional to the initial slope inclination and to the maximum wave height. The constants of proportionality and the power exponents depend in both cases on the soil character.

The ratio between the material carried away and that deposited at the toe depends on the coarseness of the soil, on the dimensions of the waves and the speed of their flow away from the banks, on the degree of the initial saturation of soil and on the initial bank inclination. In predicting this ratio, the observations corresponding to similar local conditions have to be taken into account.

L. Suklje, Yugoslavia

4177. Turzynski, L. D., Groups of piles under mono-planar forces, *Struct. Engr.* **38**, 9, 286–293, Sept. 1960.

4178. Samoilov, V. P., The forces arising in the penetration of soil by shield edges or force-driven pipes (in Russian), *Vodosnabzhenije i San. Tekhnika* no. 10, 19–26, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4309.

The inconsistency of a number of current views on this subject is pointed out. The force required for penetration into the soil is recommended to be calculated by the formula

$$P_{\Pi} = P_K + P + P_{B.\Pi},$$

where P_K is the force of penetration into the soil, of the blunted, cutting edge; P the force required to bring the soil within the cutting part in a plastic state; $P_{B.\Pi}$ the force required to overcome the friction of the soil along the internal surface of the head (cutting ring) of the shield. The force $P_{B.\Pi}$ may be disregarded if the soil is continuously removed at a distance from the entry cross section (in the direction of the excavated tunnel), not exceeding 0.15D of the cutting edge. In a first approximation, the force P can be determined by solving the axially-symmetrical problem of the penetration of the cutting edge into an imponderable, friable medium. Applying the relationship $P'_{o(oc)} = P'_{o}(\Pi\pi)\cdot k$, between the magnitude of the unit force of penetration into an imponderable medium of two-dimensional ($P'_{o}(\Pi\pi)$) and circular ($P'_{o(oc)}$) cutting edges (correspondingly, in conditions of the symmetrical and axially-symmetrical problem), the author reduces the determination of the force P to solution of the corresponding, plane-symmetrical problem. For the latter, the method of graphical analysis of S. S. Golushkevich is used. The force P_K of penetration of the cutting edge is determined by the expression for the two-dimensional problem of the stability of a foundation:

$$P_K = \Pi\delta_K \left[(\sigma_H + k \operatorname{ctg} \rho)\frac{1+\sin\rho}{1-\sin\rho}\exp \pi \operatorname{tg}\rho - k \operatorname{ctg}\rho\right]$$

where $\Pi = \tau D$. The author illustrates the described method of calculating the total force P_{Π} by a concrete example.

I. Govyadinov

Courtesy Referativnyi Zhurnal, USSR

4179. Berezantsev, V. G., Yaroshenko, V. A., Prokopovich, A. G., Razorenov, I. F., and Sidorov, N. N., An investigation into the stability of sand foundations (in Russian), *Trudi Vses. Nauk-i. In-ta Transp. Str-va* no. 28, 140 pp., 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4289.

Results are presented of experimental and theoretical investigations on the nature of the deformations and the conditions for the attainment of the limiting state in the case of sand foundations according to their strength, for different relationships between the depth of the foundation and the density of the sand. Analysis of the experimental data has enabled two essential phases in the deformation of sand foundations to be isolated. The first (initial) phase begins with the instant of application of the load, and ends with formation of a compacted core. The foundation undergoes a process of compression of the soil and formation of local slip planes (discontinuities). A compacted core is formed under the load.

The second phase extends from the instant of formation of this compacted core. The foundation experiences a violent development of slip in large bodies of soil along continuous slip planes. This phase is accompanied by considerable and rapidly growing settling phenomena and lateral displacements of the foundation, presenting danger to the normal existence of the structure. The limiting strength condition of a sand foundation is reached, therefore, at the transition from the first deformation stage to the second. Depending on the density of the sand and the relative sinking of the foundation, four characteristic forms of the limiting state can be distinguished: in sands of high and medium density, when the (building) foundation is erected on the surface or only inconsider-

ably sunk ($0 \leqslant b_s/b \leqslant 0.5$; Case 1a), as also at slightly greater depths ($0.5 \leqslant b_s/b \leqslant 1.0 \div 1.5$; Case 1b), the soil adjoining the foundation is bulged along continuous slip planes; in loose sands, whatever the foundation depth, and in dense and medium-density soils in the case of deep foundations, there is an interaction between the regions of slip and the regions of compaction in the soil, located above ($1 \div 1.5 \leqslant b_s/b \leqslant 3 \div 4$; Case 2a) or below ($b_s/b > 3 \div 4$; Case 2b) the sole of the foundation. In the authors' opinion, the improvement in the method of strength analysis of such foundations should, correspondingly, consist in determining the character of the possible limiting strength condition for sand of a particular density, and a particular depth of foundation. Determination of the resistance of the soil foundation then proceeds by the usual methods of analysis for a limiting state. Analysis of foundations in this manner, according to particular deformation characteristics and conditions for the attainment of a limiting state has, in the cases enumerated, shown good agreement with the experimental results, and disclosed high reserves in the carrying capacity of foundation soils.

A. I. Govyadinov

Courtesy Referativnyi Zhurnal, USSR

4180. Glushkov, A. T., An experimental determination of the contact pressures at the interface of a driven element and an elastically-yielding foundation (in Russian), Dokladf Mezhvuz. Konferentsii po Ispytaniam Sooruzh., Leningrad, 1958, 137–146; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4290.

Experimental results are presented on the driving of rigid and flexible elements into a sand foundation, made in a three-dimensional tray. The contact pressures were measured by membrane boxes of the author's own construction. The investigations extended to the influence of the moisture content of the sand on the nature of the pressure distribution under foundations of varying form and rigidity. According to the author's experimental results, the character of the pressure distribution depends on the cohesion of the foundation, and not on the mean value of the pressure on the foundation, or the extent of the load-transmitting area. Tests with an impressed beam in conditions of symmetrical loading have shown an irregular distribution of the pressure in width: according to a parabolic law for dry sand, and anticlinal (saddle-shaped) in moist sand. For conditions of asymmetrical loading, the critical values have been determined for the eccentricities of the points of application of the concentrated force. It is demonstrated that in analytical calculations it is necessary to take account of the development of regions of plastic deformation under the ends of the beam. A comparison is made between the experimental results and analytical calculations following the theory of an elastic semi-space, using a special, foundation model. The author concludes the suitability of using a composite model for the calculation of foundations on real soils.

L. P. Vinokurov

Courtesy Referativnyi Zhurnal, USSR

4181. Trifonov-Yakovlev, D. A., The compacting of sandy soils by pulsatory action (in Russian), *Trudi Nauk-i. Sektora Mosk., Fil. In-ta "Orgenergostroi"* no. 1, 72–85, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4310.

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break off at the extreme edge where the edge is so thin that the material cannot stand the impact of grinding wheel grains."

The tool force diagram is thus significantly changed to include forces along the rounded edge. In turn, this changes magnitudes of the cutting force, thrust force, and better correlates tool-chip interface friction coefficients with established values. Even chip curling is claimed to be, thereby, better explained.

An experimental procedure for separating ploughing forces from other cutting forces is described. Data so obtained by the author are presented in substantiation of theory and conclusions reached.

J. P. Vidosic, USA

4185. Bagley, F. L., Jr., and Mennell, R., Machining characteristics of leaded steel, *ASME Trans.* **82** D (*J. Basic Engng.*), 2, 347–359, June 1960.

In this paper, two kinds of information were revealed: First, the mechanism by which lead in steel affects the cutting process and, second, machining data of leaded steel at higher cutting speeds and higher hardness levels. To get the first type of information, orthogonal cutting was used; while for the second type of information, tool-life testing was relied upon.

In orthogonal cutting, differences between leaded and nonleaded steels were found at low speeds for all hardness of work metals. The cutting force, coefficient of friction in cutting, shear angle and shear energy all showed differences below the same critical speed for the same hardness of the work piece.

From tool-life testing, it was found that the leaded steel has a greater slope in tool-life plots. That is, an increase in cutting speed tends to decrease the tool life more for leaded than nonleaded steels.

Different theories which could explain the effect of addition of lead into steel were cited. It was concluded that the best mechanism to that effect is attributed to the fact that lead additive acts as a lubricant on the chip-tool interface below a certain critical speed. This speed is directly related to the tool temperature as the author proposed.

C. T. Yang, USA

4186. Okushima, K., and Hitomi, K., Plastic flow of chip in steel cutting, *Bull. JSME* **3**, 12, 556–560, Nov. 1960.

Authors discuss primary deformation in flow region and secondary deformation due to friction between chip and face of tool. Orthogonal cutting tests of steel show cutting ratio, direction of flow, roughness of chip under various cutting conditions and indicate existence of flow region instead of a single shear plane assumed in conventional theory.

G. G. Meyerhof, Canada

4187. Nekrasov, S. S., The load distribution on the front edge of a cutter working in brittle materials (in Russian), *Nauchn. Trudi Mosk. Gorn. In-ta* no. 19, 111–132, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4270.

A description of test made by strain-optical methods, concerning the pressure distribution under a rigid punch pressed into an elastic medium (semiplane, quarter-plane, and quarter-plane with a boundary cutout). In all cases, the pressure distributions under the punch are very similar. The author points out that this circumstance confirms his considerations on the nature of the pressure distribution on the front edge of a cutter working in brittle materials.

G. S. Shapiro

Courtesy Referativnyi Zhurnal, USSR

4188. Looman, J., Side and end hobbing cutters for generating stepped gears with special tooth systems (in German), *ZVDI* **103**, 6, 222–228, Feb. 1961.

Complicated hobs are required to generate on a large-batch basis spur gears having a hypoid or one-sided cycloid tooth system. It is recommended that simple side or end cutters be used for the manufacture of experimental gears and for small lot production duties. The dimensional design of the profiles of such cutters is explained with the aid of three detailed examples.

From author's summary

4189. Rice, W. B., The formation of continuous chips in metal cutting, *Engng. J., Montreal* **44**, 2, 41–45, Feb. 1961.

Results of experimental investigations into the formation of continuous chips in metal cutting are presented. Observation of these chips using high-speed photography is also described.

From author's summary

4190. Kobayashi, S., and Thomsen, E. G., The role of friction in metal cutting, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 324–332, Nov. 1960.

Metal-cutting studies were made with free-cutting steel SAE 1112 and alloy steel SAE 4135 in the as-received condition with artificially controlled tool-chip contact areas and flank contact areas (artificial wear lands). The experimental results for steel SAE 1112, at a speed range of 0.083 to 1010 fpm, reveal that friction under metal-cutting conditions on the rake face can be explained satisfactorily by a junction model with possible superimposed general plastic flow above the junctions in accordance with the general rules of plastic deformation (von Mises instantaneous yield criterion). The experimental results also reveal that the friction mechanism at the controlled flank-wear contact area is essentially the same as that occurring at the tool face. The difference in behavior of the two steels was attributed to the role that manganese sulfide appears to play in free-cutting steels.

From authors' summary

4191. Chang, S. S., and Heginbotham, W. B., Comparisons between the shearing properties of alpha-brass as derived from the cutting process and from static and impact torsion tests, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 315–323, Nov. 1960.

In the work presented here direct comparisons were made between the results obtained by analyzing the cutting process and the results of both static and impact torsion tests on the same material. The material used was alpha-brass (70 to 30 per cent Cu-Zn). Choice rested on this material because it appeared to exhibit a very large internal frictional coefficient if results obtained from an "orthogonal" cutting test were analyzed in terms of the now familiar Merchant analysis. These cutting tests indicated that the sum $2\phi + \tau - \alpha = 67$ deg ± 5 per cent if the correct cutting conditions were chosen. The difference between 67 deg and the ideal figure of 90 deg, which should be the case if the material exhibits no Bridgman effect, should therefore be easy to find. It is shown that this difference could not be accounted for by the internal frictional coefficient effect. Due to the failure to explain the difference, efforts were shifted toward an evaluation of the possible effects of rate of strain on the stress-strain curve for the material. Toward this end a special impact-torsion machine was designed capable of impressing strain rates of 1000 per second abruptly to a tubular specimen. This figure for strain rates compares favorably with the lower limit of the estimates of strain rates in metal cutting by another investigator. Photomicrographs covering the range of conditions tested were produced by an abrupt stopping technique and from these it was possible to explain certain variations, particularly in the constant $2\phi + \tau - \alpha = C$. It was not found possible to make any estimate of strain rates from the photomicrographs.

From authors' summary

4192. Dovnorvich, V. I., The three-dimensional problem of a rigid die of circular plan, the surface whereof represents a particular polynomial in orthogonal coordinates (in Russian), *Ucb. Zap. Belorussk. In-ta Inzb. Zb.-D. Transp.* no. 2, 31–46, 1958; *Ref. Zb. Mekb.* no. 4, 1959, Rev. 4108.

A solution is given for the impression of an unsymmetrical rigid die, the equation of the face of which can be represented by a polynomial. The integral equation of the problem is reduced to the form of a linear system of algebraic equations.

M. Ya Leonov
Courtesy Referativnyi Zhurnal, USSR

Fracture (Including Fatigue)

(See also Revs. 4037, 4041, 4045, 4112, 4184, 4216, 4220, 4270)

4193. Williams, M. L., The bending stress distribution at the base of a stationary crack, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 78–82, Mar. 1961.

Paper, extending an earlier one dealing with extensional stress distributions, theoretically studies stresses around a crack due to bending loads. It is found that the stresses possess inverse square-root singularity in terms of distance from crack point, and in the crack direction symmetric principal stresses are of opposite sign and in the ratio $(1 - \nu)/(3 + \nu)$ while in the extensional case they are identically equal. This leads to the observation that more yielding might be expected as the percentage of bending to extensional stress increases; results are also discussed in connection with combined extensional and bending loading.

From author's summary by A. F. W. Langford, So. Australia

4194. Kies, J. A., Smith, H. L., and Irwin, G. R., Fracture mechanics and its application to engineering problems (in French), *Rev. Métall.* **57**, 2, 101–117, Feb. 1960.

Paper is a report on a lecture given by the authors at the autumn meeting of the Société Française de Métallurgie in 1958. It outlines authors' ideas and investigations on fracture propagation, stress analysis, crack driving force, crack toughness and crack mode. An original paper of these authors is reviewed in AMR **13** (1960), Rev. 4558.

A. Kochendorfer, Germany

4195. Frisch, J., Fracture of flat and curved aluminum sheets with stiffeners parallel to the crack, *ASME Trans.* **83** D (*J. Basic Engng.*), 1, 32–38, Mar. 1961.

The mode of crack propagation and failure in relatively large 2024-T3 aluminum sheets reinforced with stiffeners parallel to the crack direction has been investigated. Curved specimens with a 69-in. radius of curvature as well as flat panels were subjected to uniaxial tension perpendicular to a simulated crack to study the effects of curvature, crack location, and stiffener spacing. Increase in strength due to stiffening particularly in the curved panels was observed although these specimens exhibited considerable lower crack strength than flat ones. For the specimens tested, crack location as well as variations of stiffener spacing from 3 to 12 in. had no appreciable effect on either critical crack length or failure stress.

From author's summary by T. J. Dolan, USA

4196. Ludley, J. H., and Drucker, D. C., Size effect in brittle fracture of notched E-steel plates in tension, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 137–139 (Brief Notes), Mar. 1961.

Much of authors' experimental program on the initiation of brittle fracture in ordinary structural steel plates has been based strongly on the assumption that a Griffith type of theory is not applicable. Direct experimental evidence is presented here to support that preconceived notion.

From authors' summary

4197. Ryder, R. H., Prediction of unstable crack length in aluminum alloys, Coll. Aero. Cranfield, Note 109, 3 pp. + figs., Oct. 1960.

A method was set down for predicting the unstable length of a crack in a flat sheet of aluminum alloy subjected to a steady tensile stress.

The basis of the method was to take the work done to failure in the "neck" region of a tensile test specimen and apply it, with a suitable constraint factor, to the flat sheet to give the work rate required to propagate the crack.

Experimental evidence is produced in support of the method.

From author's summary

4198. Cornet, I., and Grassi, R. C., A study of theories of fracture under combined stresses, *ASME Trans.* **83** D (*J. Basic Engng.*), 1, 39–44, Mar. 1961.

Experimental data are presented for a cast nodular iron and for high-silicon cast iron materials which represent limiting conditions of ductility in a test of fracture theories. Data of other pertinent investigations are reviewed and various failure theories are discussed with regard to their applicability. It is concluded that failure under combined stresses of brittle materials can be predicted adequately by applying a notch modified distortion energy criterion.

From authors' summary by T. J. Dolan, USA

4199. Kinard, W. H., and Collins, R. D., Jr., An investigation of high-velocity impact cratering into nonmetallic targets and correlation of penetration data for metallic and nonmetallic targets, NASA TN D-726, 31 pp., Feb. 1961.

Experimental results have been obtained on the cratering of metals and nonmetals at velocities varying from 500 feet per second to 20,000 feet per second with various combinations of metallic and nonmetallic targets and projectiles. Materials investigated include nylon, graphite, laminated phenolic resin, and aluminum. From a study of nonmetallic targets after impact, it is indicated that the craters were formed by the crushing of and displacement of the target material. An equation for the relationship between penetration and momentum per unit area has been modified to predict the penetration of target materials tested within the range of this investigation.

From authors' summary

4200. Kinard, W. H., and Collins, R. D., Jr., A technique for obtaining hypervelocity impact data by using the relative velocities of two projectiles, NASA TN D-724, 14 pp., Feb. 1961.

A facility is described which uses the relative velocity of projectiles from two facing guns to obtain hypervelocity impact data. A 22-caliber light-gas gun is used to launch projectiles toward targets fired from a 37-millimeter powder gun. Results of several preliminary firings are included for an impact-velocity range from 12,050 feet per second to 21,850 feet per second.

From authors' summary

4201. Takeda, Yu., Minamisawa, C., and Tokutake, M., Observations on the fatigue damage of copper, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 47–51, 1960.

4202. Kawamoto, M., and Seki, M., Some experiments on fatigue under multiple repeated stresses, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 15–19, 1960.

4203. Endo, K., Some experiments on the effect of rest on stressing during high temperature fatigue, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 64–67, 1960.

4204. Taira, S., and Ohnami, M., Creep under rapid cyclic temperatures, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 77–80, 1960.

Experimental Stress Analysis

(See also Revs. 4005, 4035, 4044, 4056, 4066, 4147, 4183, 4194, 4247)

4205. Durelli, A. J., and Daniel, I. M., Structural model analysis by means of moire fringes, *Proc. Amer. Soc. Civ. Engrs.* **86,** ST 12 (*J. Struct. Div.*), 93–102, Dec. 1960.

The objective of this paper is to demonstrate the use of moiré fringes in the measurement of displacements and rotations in structural models. The method, applied to the cases of a simply supported beam, a continuous beam, and two plane frames, gave results in satisfactory agreement with theory, whenever such comparison was made. The techniques used are extremely simple and inexpensive. From authors' summary

4206. Tramposch, H., and Gerard, G., An exploratory study of three-dimensional photothermoelasticity, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 35–40, Mar. 1961.

A sandwich technique which utilizes an embedded polariscope consisting of two sheets of polarizing material cemented within a plastic model was evaluated for application to three-dimensional photothermoelasticity. The evaluation included strength tests of cemented joints, photoelastic tests of simple bending models and a sphere, all under mechanical loading. The sandwich technique was then applied to thermal-stress problems associated with a thick-walled cylinder under steady-state conditions. The experimental results correlate well with theory. The results obtained indicate the general applicability of the sandwich technique to three-dimensional stress problems generated by mechanical or thermal loads...

From authors' summary by E. E. Sechler, USA

4207. Tramposch, H., and Gerard, G., Correlation of theoretical and photothermoelastic results on thermal stresses in idealized wing structures, *ASME Trans.* **82** E (*J. Appl. Mech.*), 1, 79–86, Mar. 1960.

Paper presents experimental results obtained by photothermoelastic method on two idealized profiles representing wing structures and intended to simulate problem of aerodynamic heating of high-speed airfoil. Sections tested were I-beam and two-cell beam constructed of epoxy resin. Temperature distributions were also obtained on stainless-steel single-cell beam. Thermal loading was produced by applying dry ice to flanges, thus obtaining step-function temperature change. Temperatures were measured by thermocouples located at midsection.

Experimental results were correlated with three theories. First theory assumes uniform temperature in flanges and no heat transfer to web; second theory also assumes uniform temperature in flanges but included heat transfer to web; third theory, constructed by authors from standard solution, assumes temperature variation through flange in direction of thickness. Best correlation was obtained with third theory. Computed results show no significant difference between first two theories, thus showing that heat transfer to web may be disregarded when web is thin.

J. E. Goldberg, USA

4208. Flynn, P. D., and Frocht, M. M., On the photoelastic separation of principal stresses under dynamic conditions by oblique incidence, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 144–145 (Brief Notes), Mar. 1961.

4209. Usui, E., and Takeyama, H., A photoelastic analysis of machining stresses, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 303–308, Nov. 1960.

Direct measurements of the distributions of normal and frictional stresses on a rake face under cutting conditions have been considered to be practically impossible. However, as reported in this paper, the stress distributions have been successfully obtained photoelastically by using a tool made of a photoelastic material.

According to the authors' experiment, the frictional stress on the rake face is distributed uniformly over a wide range of the tool-chip contact length, but it decreases rapidly near the point of chip-separation on the rake face. As to the normal stress, it has a peak near the cutting edge, being rather stationary in the middle part of the contact length and decreasing gradually toward the point of chip-separation. From authors' summary

4210. Guerrini, B., Experimental investigation of the stress distribution in rectilinear cantilever beams, loaded by bending moment and shearing stress (in Italian), *Aerotecnica* **40,** 4, 231–240, Aug. 1960.

The stress distribution in rectilinear cantilever beams of constant thickness, loaded by bending moment and shearing stress, has been determined by photoelasticity.

Diagrams have been made which give maximum shearing stress variation as a function of the variables having influence over the stress distribution in the examined element.

From author's summary

4211. Blokh, V. I., Some experimental methods in the theory of elasticity (in Russian), *Trudi Kbar'kovok. Avtomob.-Dor. In-ta* no. 21, 81–98, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4613.

Some experimental methods of stress determination are examined, founded on the analogies between the theory of elasticity and problems in other fields of physical research. The fundamental propositions of the membrane theory of torsion are recapitulated. An experimental installation is described, with which the checkerboard image reflected from a distorted soap film, stretched over a test area, has been photographed. Formulas and charts used in evaluating the above photographs are also presented. A description is also given of the electrodynamic analog of torsion. According to this analogy, the torsional stress function resembles the electrical potential distribution function in a flat conductor of the same shape as the cross section of a twisted bar, on the condition that the potential on the boundary follows some particular known law. The experimental installation is described. The experimental, two-dimensional conductor was represented by a weak electrolyte solution, contained in a shallow wax dish. The boundary of the experimental field was made in the form of a thin, perforated partition behind which a strong electrolyte was contained in the dish; this acts as a potentiometer and ensures the required potential distribution in the conductor field. The potentials at points inside the experimental field are measured by means of a feeler connected to a resistance bridge. Finally, the laminar analog of the two-dimensional problem of the theory of elasticity is described. By this method the stress distribution is found by measuring the bending deflections of a thin, elastic plate (lamina), of the same outline as the experimental field, and distorted along the edge according to the predetermined loading diagram of a flat, elastic body. The fundamental propositions of the method of laminar analogy are laid down, and it is shown that in certain cases the method is to be preferred to optical methods of stress analysis. S. V. Boyarshinov

Courtesy Referativnyi Zhurnal, USSR

Material Test Techniques

(See also Revs. 4218, 4219, 4220)

Book—4212. Dietz, A. G. H., and Eirich, F. R. (Co-chairmen), High speed testing, Vol. 1 (Symposium held at Boston, Dec. 8, 1958), New York, Interscience Publishers, 1960, vii + 112 pp. $5.

The book compiles eight separate technical papers contributed by eleven authors at a symposium held in Boston in 1958, and deals primarily with the effects of rate of straining on mechanical properties of a wide variety of materials. In general, most of the papers emphasize instrumentation, equipment and experimental test techniques as applied to determination of mechanical properties of metals, plastics, yarn, and solid rocket fuel. All of the contributions are concerned primarily with experimental observations of the effects of temperature and strain rate on the mechanical behavior of materials. The first paper reports high-speed tensile tests of thermoplastics employing an Instron and a special high-speed machine. Ultimate properties and elastic moduli are reported for tests ranging from static conditions to approximately impact and for a wide range of temperatures. The second paper describes a tension tester for measuring behavior of plastics with grip displacement rates of 20 to 5000 in./in./min; tension data are reported for various strain rates and for several plastics materials. The third paper discusses the tensile properties of several types of solid rocket fuel compositions over wide ranges of temperature and strain rates. The data indicate that properties of certain types of elastomeric compositions tend to improve as testing speed increases while others pass through maximum values and then degrade with further increase in test speed.

The fourth paper on impact testing of package cushioning materials describes apparatus for dynamically determining the required cushion characteristics to protect a fragile item under shock loadings. The fifth paper on rapid impact loading of textile yarns measures tensile properties at impact velocities up to 70 meters per second with transverse or longitudinal impact. Above 50 meters per second strain wave-propagation phenomena become appreciable and properties depend upon multiple reflections of strain waves in the yarn specimen. At impact speeds of the order of 250 meters per second, the critical velocity is reached for which specimens tend to break immediately upon longitudinal impact. The sixth paper presents an analysis of the stress relaxation of viscoelastic materials which are undergoing extension at constant rate. It is shown that the box distribution of relaxation times can be used to describe the behavior of fibers, and parameters of the box distribution are determined from constant strain rate data. The seventh paper presents experimental data on the true stress-true strain properties of commercially pure titanium in tests conducted from +800° to −319° F at 0.001 to 0.1 min^{-1}, additional tests were conducted at 3×10^3 min^{-1} at room temperature, and at 40 min^{-1} at −319° F. The data show that the strain at maximum load increases with decreasing temperature and decreases with increasing strain rate. The eighth paper discusses studies of metals at ballistic rates of loading and outlines a technique for quantitive investigation of the scabbing phenomenon. Preliminary experiments on aluminum specimens indicated a stress for fracture of approximately 240,000 psi.

T. J. Dolan, USA

4213. Hockett, J. E., Compression testing at constant true strain rates, *Proc. Amer. Soc. Test. Mat.* **59,** 1309–1319, 1959.

A cam plastometer was used to conduct constant true strain rate compression tests of: (1) commercially pure aluminum at room temperature with true strain rates of 2×10^{-1}, 4.6×10^{-3}, and 1.5 per second; (2) depleted uranium at 600, 500, 400 and 300 C with true strain rates of 10^{-3}, 10^{-1} and 1 per second.

Statistical methods were employed to compare the experimental data with an empirical equation relating true stress to true strain. This empirical equation contains three parameters, two of which are dependent upon both strain rate and temperature, whereas the third parameter is essentially independent of temperature. Variance and standard deviation of the fit of the curve and of three parameters were determined.

Photographs and some details of the equipment used are given. Appendix discusses statistical methods employed in curve fitting and computing variances and standard deviations.

W. K. Rey, USA

4214. Girard, F., and Vidal, G., A new method for measuring at high temperatures Poisson's ratio of metals and alloys (in French), Revue de Metallurgie **57,** 2, 118–124, Feb. 1960.

To avoid at high temperatures the inaccuracy of static experiments or the difficulty of exciting two modes of vibration simultaneously, authors have developed a method based on the fact that the frequency of longitudinal oscillations of cylindrical bars depends on the ratio diameter d/length l and is influenced by both Young's modulus E and Poisson's ratio σ. These quantities are determined by testing two specimens with $d/l = 1/5$ and $= 1$. Designs for low and high temperatures are described. Experimental errors in σ are estimated to be less than 1%. Results for several metals and alloys are given. On the whole σ increases with increasing temperature, whereas E decreases, but structural changes lead to irregularities. Method seems to be very simple but nevertheless highly accurate.

A. Kochendorfer, Germany

4215. Neppiras, E. A., Techniques and equipment for fatigue testing at very high frequencies, *Proc. Amer. Soc. Test. Mat.* **59,** 691–710, 1959.

Paper describes ultrasonic vibrators for fatigue tests. Vibrators consist of three resonant sections: transducer, velocity transformer, test specimen. Three types of electromechanical transducer are discussed: piezomagnetic, piezoelectric and electrodynamic. A systematic and detailed survey of design requirements for vibrators is presented, especially for axially loaded specimens and piezomagnetic (magnetostriction) transducers, allowing test frequencies up to 50,000 cps. Internal heating and cooling of specimens are discussed. Among applications mentioned are research on fatigue at very high endurances, dynamic tests on nonmetallic materials and adhesive bonds. A frequency effect on the fatigue limit is noticed.

J. Schijve, Holland

4216. Enomoto, N., A method for determining the fatigue limit of metals by means of stepwise load increase test, *Proc. Amer. Soc. Test. Mat.* **59,** 711–722, 1959.

Author criticizes the Prot method for inefficiency in saving time and specimens and for complex loading method. A stepwise load increase is advocated and for two carbon steels he shows by tests that the same failure stresses are obtained as with a linearly increasing load. Main point of paper is the proposal to perform tests with progressive loading at one rate of loading only and to obtain the fatigue limit by reducing the failure stress by a certain percentage. Suggestions for the magnitude of the loading rate and the reduction percentage are presented based on literature data. Additional experimental verification is recommended.

J. Schijve, Holland

4217. Upton, P. B. G., and Bromley, R. H., Some aspects of the Schiefer abrasion machine, *Wear* **3,** 5, 388–393, Sept./Oct. 1960.

The design of the Schiefer machine and some aspects of the action of a machine of this general pattern are considered. A possible theoretical improvement in design to permit more uniform rubbing with a wider variety of abradants is suggested.

From authors' summary by M. B. Hollander, USA

Properties of Engineering Materials

(See also Revs. 4039, 4042, 4045, 4112, 4147, 4191, 4194, 4198, 4212, 4213, 4234, 4399, 4501, 4536)

4218. Espey, G. B., Repko, A. J., and Brown, W. F., Jr., Effect of cold rolling and stress relief on the sharp edge notch tensile characteristics of austenitic stainless steel sheet alloys, *Proc. Amer. Soc. Test. Mat.* **59,** 816–836, 1959.

The cold-reduced up to 70% AISI 301 and AISI 304L sheet steels were tested at room and −320°F temperatures. The stress relief temperatures 400°F to 1100°F were then applied to 70% cold-reduced AISI 304L and 60% cold-reduced AISI 301 steels. The smooth and sharp notch tensile properties are given for 0.063-in. gauge specimens. Only the one heat of each steel was used during investigations. The discussion covers the influence of structural changes in steels on their mechanical properties for both directions of sheet. The general conclusion seems to be that the lowest stress relief temperatures are of higher practical value. The remaining conclusions apertain or could be useful in specific cases of design or manufacture.

While paper is interesting and useful, it is considerably less clean cut in presentation and conclusions than the following paper. It is difficult to point out the main practical feature of the experiments. W. De O'Byrn, England

4219. Espey, G. B., Jones, M. H., and Brown, W. F., Jr., The sharp edge notch tensile strength of several high-strength steel sheet alloys, *Proc. Amer. Soc. Test. Mat.* **59,** 837–884, 1959.

The high tensile sheet steels were tested at room temperature for smooth and sharp notch tensile properties after tempering at variable temperature or using variable amount of cold reduction. The steels were (1) ferritic: (a) low alloyed SAE 4340, AMS 6434, UHS 280, 300M, X-200, (b) hot-work tool steel Vascojet 1000, (c) martensitic stainless AISI 410, 12MoV; (2) precipitation-hardening stainless AM 350, 17-7PH, PH15-7Mo; (3) cold-worked austenitic stainless AISI 304L, AISI 301. Tested steels were of 0.063-in. gauge except SAE 4340 and one heat of AMS 6434 which were 0.095-in. Only one heat of each steel was tested except AMS 6434 which had two different heats used but of different gauge and third heat was vacuum melted. The conditions of experiments are quoted in detail. The discussion of results and relative merits of steels is far reaching and closely interwoven with results published by other investigators. The most important conclusions are the necessity of correlation (1) between smooth tensile and sharp notch tensile properties and (2) between fault which is represented by sharp notch test and actual faults which could be found in finished components. The relative merits of different notch ductility tests are discussed.

Paper is very good. The problem and experiments are presented to the smallest detail. The results are discussed fully and interesting practical conclusions are drawn. The only handicap is that only one heat of each steel was tested and, as is pointed out by authors, there is no certainty that results could not be altered by variations in chemical composition or by physical variations of melt. W. De O'Byrn, England

4220. Folgar, R. P., Fatigue tests on three cast irons at elevated temperatures, *Proc. Amer. Soc. Test. Mat.* **59,** 767–773, 1959.

Paper reports results of pulsating and reversed push-pull fatigue tests on three cast irons, over the temperature range 700–1000° F, using the staircase method for a life of 10^7 cycles.

C. E. Turner, England

4221. Brady, R. R., AISI type 304L stainless steel with improved strength, *Proc. Amer. Soc. Test. Mat.* **59,** 774–785, 1959.

Paper reports studies to increase strength of a low-carbon austenitic stainless steel (18% or 9% Ni type) by increasing the nitrogen content, without impairing the intergranular corrosion resistance.

Tensile test data are presented up to 1200°F and creep data up to 1500°F. C. E. Turner, England

4222. Agerman, E., Notch sensitivity in steel, *Acta Polytech. Scandinavica* no. 288 (Mechanical Engng. Series no. 8), 44 pp., 1960.

The present paper gives an account of an attempt made to compare various formulas for calculating the notch effect resulting from stress concentrations in the form of notches, fillets and holes. In order to make this comparison impartial, it has been performed with the aid of statistics. Since the data forming the basis of the theories and the investigation are empirical, the result cannot naturally be exact. The testing machines yield certain errors, and this must be considered. The accuracy of each of the theories is given in the form of a mean value and a dispersion. When evaluating the theories, preference is given to that one having the smallest dispersion. From author's summary

4223. Endo, T., Strength of steels under repeated impulsive load, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 20–23, 1960.

4224. Digges, T. G., and Rosenberg, S. J., Heat treatment and properties of iron and steel, Nat. Bur. Stands. Monogr. 18, 40 pp., Oct. 1960.

4225. Lord, J. B., Mechanical properties of solid and porous stainless steel sheet material at elevated temperatures, Coll. Aero. Cranfield, Note 107, 7 pp. + charts, Oct. 1960.

4226. Corruccini, R. J., and Gniewek, J. J., Specific heats and enthalpies of technical solids at low temperatures, Nat. Bur. Stands. Monogr. 21, 20 pp., Oct. 1960.

4227. Jackson, E. G., Muench, C. F., and Scott, E. H., Evaluation of gear materials scoring at 700 F, *ASLE Trans.* **3,** 1, 69–82, Apr. 1960.

4228. Gol'dfarb, V. M., The influence of nonhomogeneity of polycrystalline materials on their mechanical properties (in Russian), *Ucb. Zap. Latv. In-ta* **20,** 261–272; 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10974.

A model is proposed for the study of the qualitative analysis of the stresses and deformations in a polycrystalline medium; this model takes the form of a laminar system with identical orientation of the crystalline lattice for a given lamina but with different orientation for different laminae. A model of this type of polycrystal during tension in a direction perpendicular to the laminae is the equivalent of the known arrangement of successive conjunction of samples which during tension in a direction coinciding with the direction of the laminae becomes an arrangement of parallel conjunction. In the first case the stresses in the samples are identical, in the second the deformations. The laminar system generalizes directly the given arrangements in their simplest form, enabling the volumetric macro-stressed state to be reproduced. In addition it differs from the polycrystalline medium by the fact that the "grains" in the laminar structure differ essentially as regards form and dimensions from the grains of polycrystalline metals. The laminar system enables qualitative modelling to be accomplished of the properties of polycrystals such as the volume of the macro-stressed state, during the single-axis macrostressed state (of the first order), the presence of distribution of stresses and deformations in the layers (grains) in relation to the mean or macro-

scopic stresses and deformations, the divergence in time of the beginning of plastic deformations in the laminae, the presence of "constrained" deformation resulting from interaction between the laminae, and so forth. The laminar system, by comparison with the more simple models of the polycrystal which bring about the parallel and successive conjunction of the samples, enables a more complete modelling to be made of the interaction between the grains of the polycrystal along their boundaries. In the general case of the polycrystalline medium, containing diverse grains both as regards form and dimensions, the interaction between the grains has a statistical character and is expressed in the form of a correlation between the components of the tensors of the microstresses (which correspond to microdeformations) in neighboring points of the medium.

S. D. Volkov

Courtesy Referativnyi Zhurnal, USSR

4229. Dwight, J. B., Aluminium strut design, *Struct. Engr.* **39,** 2, 47–61, Feb. 1961.

In the new Report on the Structural Use of Aluminium, which will be published in the near future by the Institution and which will supersede the 1950 report, recommendations are given for arriving at safe stresses in the principal types of structural members. This paper summarizes the background of knowledge that has led to the clauses dealing with struts. The three basic modes of failure (column, torsional and local buckling) are studied, and simplified procedures are presented for application to some of the more standard shapes of section.

From author's summary

4230. Pauw, A., Static modulus of elasticity of concrete as affected by density, *J. Amer. Concr. Inst.* **32,** 6, 679–687, Dec. 1960.

4231. Badziach, S., Thermo-physical properties of coals and cokes, *Brit. Coal Utilisat. Res. Assn., Mo. Bull.* **24,** 11, 485–520 Oct./Nov. 1960.

Structures: Simple

(See also Revs. 3968, 4010, 4031, 4033, 4035, 4099, 4104, 4105, 4106, 4112, 4138, 4147, 4195, 4230, 4247, 4248)

Book—4232. Bresler, B., and Lin, T. Y., Design of steel structures, New York, John Wiley & Sons, Inc., 1960, xiii + 710 pp. $9.75.

The announced objective of this book is the presentation of a rational approach to the design of steel structures and, wherever possible, to correlate this with current practice. Reviewer believes that this objective has been attained.

In addition to an excellent treatment of the usual structural design topics, the authors have included numerous additional topics, such as torsion, plastic design, high-strength bolts, light-gage members and beam-columns. In all cases the treatment is based on "The fundamental principles of structural mechanics, an understanding of the behavior of actual and idealized structures, and appreciation of practical requirements including safety, feasibility, and economy."

While presenting a fine treatment of the theoretical principles involved in design, the authors recognize that the ultimate purpose of the design is a practical structure. To this end, they have included numerous examples of the design of structural components, in addition to examples of the design of four full structures. It is felt that this book will be well received both as a textbook and as a reference.

E. R. Johnston, USA

Book—4233. Hertel, H., Leightweight construction [Leichtbau, Bauelemente, Bemessungen und Konstruktionen von Flugzeugen und anderen Leichtbauwerken], Berlin, Springer-Verlag, 1960, xxviii + 526. DM 67.50.

For use by structural engineers author gathered data on a large variety of subjects:

1. Mechanical properties of materials;
2. Strength and buckling behavior of components; rectangular plates, stiffeners, stiffened plates, sandwich plates;
3. Diffusion of concentrated longitudinal load into stiffened plates;
4. Production methods: extruding, forging;
5. Technological comments and strength data on joining methods rivets, spot-welding, glueing;
6. Some data on stress concentration factors, fatigue and crack growth;
7. A very superficial account of load conditions for aircraft.

Though author states that his book is even an extended version of his university lectures, he refrains from any attempt to develop the reader's understanding of the mechanical or physical basis and he merely describes the data available in theoretical or experimental literature by means of complex diagrams, the parameters of which are very briefly defined in the text. Stress analysis of complex structures is not considered.

Reviewer notes that too much emphasis is given to German literature, superseded as it is by later work from other sources which has not been mentioned. Given the wide variety of subjects it can hardly be avoided that some data are defective; e.g. the buckling coefficient for cylindrical shells is taken according to linear theory; warping resistance in torsional buckling of stiffeners is not considered.

The user of the book should be thoroughly acquainted with fundamentals so as to enable him to judge the value and limitations of the contents.

It is not a guide for students.

A. Van der Neut, Holland

4234. Hahn, V., and Holz, R., Analysis of the influence of creep and shrinkage on statically indeterminate steel construction with the aid of Kani's moment distribution procedure (in German), *Beton u. Stahlbeton.* **55,** 12, 274–284, Dec. 1960.

The paper is thought to be a kind of design manual, enabling the designer to perform creep calculations of complicated reinforced-concrete constructions without much introductory study. The calculations are based on Busemann's creep layer method and Kani's iteration method.

The time-dependent forces acting on a cross section are determined under consideration of compatibility conditions, but instead of elastic distortions in ordinary elastic analyses, distortions due to creep and shrinkage are considered. Before using Kani's iteration method when calculating displacements and stresses in the frame work it is necessary first to determine distortions of statically determinate beam. This is done by means of Busemann's method. Explicit expressions are given for distortion angle for different cases.

An example is given on calculation of effect of creep and shrinkage in a built-in portal frame.

E. R. Steneroth, Sweden

4235. Chocos, G. P., and Scalzi, J. B., Ultimate strength of a folded plate structure, *J. Amer. Concr. Inst.* **32,** 8, 965–971, Feb. 1961.

Objectives of this investigation were to determine the behavior of the folded plate structure as a simple beam with an irregular cross section and to verify the ultimate moment capacity by the rectangular stress block method. For this type of structure the ultimate collapse load agreed with the theoretical load within 1.8 per cent.

From authors' summary

4236. Saelman, B., A note on the minimum-weight design of spherical and cylindrical pressure surfaces, *J. Aerospace Sci.* **28,** 1, 72–73 (Readers' Forum), Jan. 1961.

4237. Young, L. E., Simplifying ultimate flexural theory by maximizing the moment of the stress block, *J. Amer. Concr. Inst.* **32,** 5, 549–556, Nov. 1960.

A method of simplifying ultimate flexural analysis is described. The stress block is defined by a method of maximizing the moment of the stress block about the neutral axis. Three separate stress functions are used to describe the stress in the concrete. The calculated type of failure and the ultimate moment are compared with results of reinforced-concrete beam tests.

From author's summary

4238. Olszak, W., Theoretical approach to problems of bound elements (in Polish), *Arch. Inzyn. Ladowej* **6,** 2, 159–202, 1960.

States of stress and strain are considered for axially symmetric elements composed of a core and a sheath (binding) having different mechanical properties. The core is made of an anisotropic material which remains linearly elastic until the ultimate stress is reached. The case of cylindrical transverse anisotropy is discussed as determined by four elastic constants if the orthotropic axes coincide with those of principal stresses. The outer sheath (the binding) is elastic-plastic.

The analysis is done for an element loaded by a longitudinal force. The increase of the load in relation to the unbound element is determined. Since the solution obtained by assuming linearly-elastic properties of the component materials does not permit an estimate of the load-carrying capacity of the element, the conditions for destruction are analyzed in addition. The possibility of reaching the limit state due to either the destruction of the core or the plastic flow of the binding is considered. To establish the condition of destruction of the core a number of experimental data are analyzed concerning the strength criteria of concrete and Mohr's limit envelope. Starting from the condition of uniform load-carrying capacity of the core and the sheath, criteria are given for optimum design of the structural elements under consideration.

The engineering advantages of bound elements are discussed.

A. Sawczuk, Poland

4239. Stevens, L. K., Direct design by limiting deformations, *Proc. Instn. Civ. Engrs.* **16,** 235–258, July 1960.

The bases of accepted methods of structural design are examined and their limitations are discussed.

Alternative criteria based on deformations are proposed for producing safe serviceable structures, and a generally applicable method is developed for satisfying these criteria. This method can be applied to both statically determinate and statically indeterminate structures which develop their load capacity mainly through the flexural rigidity of their members, and is particularly suited to rigidly-jointed frames made of ductile and strain-hardening materials.

From author's summary

4240. MacGregor, J. G., Sozen, M. A., and Siess, C. P., Effect of draped reinforcement on behavior of prestressed concrete beams, *J. Amer. Concr. Inst.* **32,** 6, 649–677, Dec. 1960.

Tests on 19 simply supported pretensioned concrete beams with draped prestressed reinforcement are described and compared with the results of tests of similar beams with straight prestressed reinforcement. The principal variables included: concrete strength, steel percentage, length of shear span, and the angle and type of drape profile. Web reinforcement was used in only five beams.

In general, it was concluded that draping the longitudinal wires did not increase either the inclined cracking load or the shear strength of the prestressed concrete beams tested. Instead, the trend of the test results indicated a reduction in both the inclined cracking load and the ultimate strength of the beams with draped wires. A comparison of the behavior of beams with draped and straight wires showed that the detrimental effect of the drape on "shear" strength could be ascribed to the earlier formation of flexural cracks in regions of combined bending and shear and the consequent earlier development of inclined cracks. For extreme combinations of the critical variables, an inclined crack occurred prior to the formation of flexural cracks in its vicinity, in which case draping the wires caused an increase in strength which could be estimated on the basis of an uncracked section analysis.

The beams with draped reinforcement required more web reinforcement to produce a flexural failure than similar beams with straight reinforcement.

From authors' summary

4241. Mastachenko, V. N., Some problems of the analysis of reinforced-concrete beams influenced by shrinkage and temperature (in Russian), *Trudi Mosk. In-ta Inzh. Zh.-d. Transp.* no. 91, 184–220, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4488.

The two-dimensional theory of elasticity is applied to the problem of the stresses arising in reinforced-concrete beams under the influence of shrinkage and temperature. The transition to the two-dimensional case is made by dissecting the reinforced-concrete beam into separate, two-dimensional elements: a steel web, four steel half-flanges, and two concrete half-flanges in the top boom. Along the section lines, the actual stresses arising there are substituted by a system of peripheral shearing stresses, represented by trigonometric series:

$$q(x) = \sum_{n=1}^{\infty} y_n^k \cos \alpha x,$$

wherein y_n^k are coefficients to be subsequently determined. These coefficients are then calculated, both from the conditions of static equilibrium and from the conditions of equality of the strains in the flanges and webs in their reciprocal interfaces. Previously, the author solved the problems of the stresses and strains in beams loaded by a peripheral, shearing force, with given strains along a part of the edges of the half-flanges. Such problems the author solved approximately, incompletely satisfying the boundary conditions, the sufficiently well-founded assumption being made that incomplete fulfilment of the conditions at the periphery has little influence on the magnitude of the stresses acting on areas perpendicular to the axis of the beam. The analytical expressions for the strains caused by shrinkage of the concrete are derived from the conditions of equality of the relative strains in the concrete component of the top flange (along the line of partition), with the corresponding strain in the steel component of the same, top member. The relative strain in the concrete is in such case represented by the algebraic sum of the deformations (strains) produced by shrinkage (setting) and the system of tangent, shearing stresses, applying along the partition line.

A similar procedure is applied to the derivation of the analytical formulas for determining the temperature stresses due to the irregular temperature distribution through the depth of the beam; the author restricts analysis to the simplest case of a stepwise curve of temperature distribution. Equations and curves are given for calculating the maximum stresses and forces in a concrete slab due to the action of shrinkage and irregular heating. A numerical example is also given for the analysis of a reinforced-concrete beam influenced by shrinkage, and a method demonstrated for approximately calculating the influence of creep of the concrete on the stress conditions in such a beam.

N. M. Figurnov

Courtesy Referativnyi Zhurnal

4242. Rando, P., Generalized method of analysis of elastic frameworks (in Italian), *G. Gen. Civ.* **98,** 9, 700–714, Sept. 1960.

Author proposes changes in a previously published procedure for analyzing frameworks with vertical columns and horizontal beams. It is valid for any number of them. Method selected takes joint rotations and translations as unknowns (slope-deflection).

Reviewer's opinion is that the word "generalized" is not correctly used because work deals with a particular case.

A. J. Bignoli, Argentina

4243. Luk'ianov, A. M., The analysis of solid two-dimensional frames (in Russian), *Nauchno-Tekn. Inform. Byul. Leningr. Politekhn. In-ta* no. 1/2, 163–171, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4353.

The problem is solved of the analysis of plane solid frames of Π-outline with rigidly constrained feet, in which the ratio of the clear height of the upright to the clear length of the cross member is constant and equal to unity, while the ratio of the height of the constant cross sections of the elements (uprights and cross member) to their lengths is assumed to be variable and equal to 1:1, 1:2, 1:3 and 1:4, successively. In all, 16 such frame structures were investigated. Two forms of frame-loading were investigated: a load uniformly distributed along the cross member, and a concentrated force applied to the cross member in the axis of symmetry of the frame. In each loading case, the weight of the frame material (concrete) was taken into consideration. The stress conditions were investigated experimentally: a grid (mesh) was applied to the surface of a model made of ordinary printers' roller composition, which was distorted by the application of the model loads. The amount of these distortions (deflexions) was measured by a comparator, after which the stresses were calculated by the formulas for the plane problem of the theory of elasticity. At the same time, the magnitudes of the normal and shearing stresses as well as the stress distribution in the cross sections of the uprights and cross bar were determined by the expressions of the theory of strength of materials. For this, the calculation span of the frame was assumed as the distance between the axes of the uprights, and its height as equal to the distance from the axis of the cross member to the line of the feet. Comparison of the experimental results with the elementary analysis has shown that such frames can be calculated by the formulas for the strength of materials only when the ratio of the height of the elements to their length does not exceed 1:4. A. A. Gorin

Courtesy Referativnyi Zhurnal, USSR

4244. Sharmazanashvili, A. Kh., The analysis of frames on an elastic foundation (in Georgian), *Sakartvelos Politeknikuri Instuti. Shromebi.* no. 9 (57), 199–216, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4354.

A description of the calculation of frames resting on an elastic foundation. Two foundation models are investigated: an elastic semi-space, and a Winkler model. In both cases, the reactive pressures are assumed to be of stepwise-varying intensity. In addition to the equation of equilibrium, use is made of the equal bending deflections of the cross member and settling deflections of the foundation. Systems of equations are set up and presented for the case of a beam supported on an elastic foundation and divided into 12 equal parts, within the limits of each of which the intensity of the reactive pressures is assumed to be constant. Two numerical examples are given. P. I. Klubin

Courtesy Referativnyi Zhurnal, USSR

4245. Kharitonov, I. P., The strength, rigidity and stability of trusses with rigid joints (in Russian), Sovershenstvovat' Tekhnol. Mashinostr. no. 1; Penza, Knigoizdat, 1957, 155–170; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4421.

The analysis is explained of combined structures of the type of crane gantries (a beam reinforced by a lattice girder), for the action of loads on and outside the joints, in the presence of rigidity and displacement of the truss joints, as well as the influence of longitudinal loading on the bending of beams, depending on the sign. In the first approximation, the longitudinal forces, displacements of the joints, and angular rotations of the truss elements are determined analytically from a hinged-bar diagram. The equations of equilibrium and method of deformation determine the angular rotations of the joints about their centers, the bending moments, and the transverse forces in the end-sections of the bars. Next, from the equations of $\Sigma x = 0$; $\Sigma y = 0$, formed for each joint, the new values of the longitudinal forces are found. The process is then repeated in the same order, the new values of the longitudinal forces being taken as the means of each two, adjacent values. It is noted that, for lattice-girder combinations, three to four approximations are usually sufficient. A numerical example is given. The satisfactory agreement between the analytical and experimental results is mentioned. The suggested method is also applicable to analysis by the limiting state of trusses with rigid joints, loaded at these joints. A. A. Gorin

Courtesy Referativnyi Zhurnal, USSR

4246. Hnevkovsky, O., Application of matrix analysis for determination of state of stress acting in plane reinforced sheets (in Czech), *Zpravodaj Vzlu* no. 5 (17), 11–26, 1959.

In this article the process of computation of internal forces acting in a plane sheet longitudinally and transversally reinforced is derived. The sheet being provided with m longitudinal and n transverse reinforcing strips is $(m-2)(n-2)$ times statically indeterminate. As statically indeterminate values, balances and linearly independent groups of forces acting in longitudinal stringers are selected, the magnitudes of which are found from the condition of minimum strain energy. The strain energy of longitudinal stringers, sheet, and transverse stringers is considered. During the derivation, the matrix analysis has been advantageously applied and proved to be very comprehensive; the numerical calculation showed some considerable advantages, enabling numerous checks. Where the inverse matrix is known, the internal forces acting in a sheet may be easily calculated. The changes in the stress pattern resulting from constructional changes may be easily determined without repeating the whole calculation. For example the state of stress around a locally reinforced cutout may be easily determined. A sheet having an asymmetrical cut-out and loaded on one end by two forces of 700 kg is given as another example. The results of experiment are in good agreement with the calculations, so that the method of computation appears to be reasonably applicable. From author's summary

Structures: Composite

(See also Revs. 4009, 4083, 4103, 4106, 4170, 4207, 4236, 4266, 4337)

4247. Kinnunen, S., and Nylander, H., Punching of concrete slabs without shear reinforcement (in English), Trans. Roy. Inst. Tech., Stockholm no. 158 (Civ. Engng. no. 3) 112 pp., 1960.

Paper is important contribution to reinforced-concrete design. Slabs supported on columns require special reinforcement around the columns. This paper obtains failing load and deformation at failure both theoretically and experimentally. Flexural reinforcements of four types—square mesh, ring, radial and combined ring-radial—are considered. Shear reinforcement will be treated in a further report. Satisfactory agreement between experiment and theory is noted. Last part of paper gives detailed design procedure. P. C. Duqne, Brazil

4248. Zienkiewicz, O. C., and Gerstner, R. W., Stress analysis and special problems of prestressed dams, *Proc. Amer. Soc. Civ. Engrs.* **87,** PO 1 (*J. Power Div.*), 7–43, Jan. 1961.

The development and use of cable methods of prestressing as applied to dams is presented. Some techniques currently in use are examined.

The problem of stress distribution within the foundation and near the base of the dams is found by superposition of known elasticity solutions with a finite difference correction. The results of the analysis are discussed and some general conclusions arrived at. A suggested method for a simplified analysis is presented.

From authors' summary

4249. Sutton, B. A., Series solutions of some surge-tank problems, *Proc. Instn. Civ. Engrs.* **16,** 225–234, July 1960.

The maximum amplitudes of the oscillations set up in a simple surge tank by sudden rejection and sudden taking on of load are obtained as infinite series in the friction factor for the pipe, assuming a friction index of 2. In the former case the effect of different values of friction index on the height of the surge is calculated.

From author's summary

4250. Davies, J. D., Bending of the walls of partly-full cylindrical tanks, *Concrete Constr. Engng.* **55,** 10, 377–379, Oct. 1960.

4251. Samu, B., Some questions of the statical computation of bus bodies (in Hungarian), *Jarmuvek Mezogasdasagi Gepek* **7,** 11, 427–432, 1960.

4252. Chuvikovskii, V. S., Local strength of ship constructions under the action of vibrational loads (in Russian), *Trudi Nauchno-Tekhn. O-va Sudostroit. Prom-sti* **7,** 2, 283–294, 1957; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10812.

A review is given of investigations relating to the problems of the strength of welded ship structures when under the action of vibrational loads. Tests of such vessels in operation showed that vibrational loads result in the appearance of cracks due to fatigue; these are to be found on the supporting edges of plates of the outer sheath (hull) and bulkheads and also where the beams of the framework pass through the plates. The problem was solved regarding the free and the forced vibrations of a plate of finite stiffness, arbitrarily fastened on long edges; it was established that the resonance curve for the vibration of the plate has the form characterizing nonlinear vibrations. It was also shown that the inelastic resistance of the material has little influence on the amplitudes of the plate's vibration. An experimental test indicated that the measured magnitudes of stresses and amplitudes up to 5–10% agree with the theoretical. The problem is solved approximately regarding the forced vibrations of a plate of finite stiffness, open on the elastic ribs and arbitrarily clamped at the ends. Formulae are obtained enabling an assessment to be made of the degree of closing (fastening) of the plates. When investigating the vibrations of the coverings (decks) the proposal is put forward to determine the coefficients of the fastening of the beams in the assembly of the coverings by reduction to the beams on the elastic base; it is established that the vibrations of the sheathing and the ribs in the majority of cases very slightly influence any change in the frequency of the vibrations of the coverings taken as a whole. Tests were carried out on special test samples working in the same conditions as the plates of the ship's hull. It was shown that the limits of strength of steels mark SKhL-I and St. 3 were practically identical; the limit of strength of joints made by two-sided discontinuous welded seams was 25% lower than for joints made by means of a continuous two-sided seam; the fatigue strength of joints made by means of a continuous single-sided seam is appreciably lower than the fatigue strength of joints made with continuous and even with discontinuous but two-sided seams. The specially carried out vibration tests indicated that at the places of passage through the plate of the stiff rib, because of the concentration of stresses in the plate at the edge of the entry of the rib, the lowering of the strength can be estimated by the coefficient 1.5 to 2.0. The comparative vibrational tests made with riveted constructions showed that the limit of strength of a two-row riveted seam is 1.5–2.0 times higher than the limit of strength of a two-sided welded joint.

V. P. Belkin

Courtesy Referativnyi Zhurnal, USSR

4253. Utkin, A. V., Vibrations of rectangular bottom plates in the engine-room section of ships (in Russian), *Trudi Gor'kovsk. Politekhn. In-ta* **14,** 1, 55–65, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10595.

The frequencies are determined of the free vibrations of a rectangular plate rigidly clamped at all its edges when the plate is subjected to assigned and constant (in regard to magnitude) loading and to compression or tensioning forces applied to the edges of the plate. The differential equation for the free vibrations is solved by the Bubnov-Galerkin method; the form for the elastic surface of the plate is taken to be a series of algebraical polynomes. As a result, simple formulas are obtained for the natural frequencies of the plate, corresponding to a varying number of semiwaves along each of its sides. Author comes to the conclusion that the error involved in the use of the approximate formulas does not exceed 4%; this result is arrived at after comparison of the special values of the frequencies calculated in accordance with the recommended formulas with the analogous values obtained from the precise computations and with the experiment carried out by the author.

V. P. Belkin

Courtesy Referativnyi Zhurnal, USSR

4254. Fatur, J., Nondimensional representation of geometrical sizes of boats (in German), *Schiffstechnik* **7,** 37, 124–130, June 1960.

4255. Becze, E. J. Aircraft structural loads, Nat. Res. Counc. Canada, Div. Mech. Engng. and Nat. Aero. Estab. Quart. Bull. 4, 1–26, Oct./Nov./Dec. 1960.

Machine Elements and Machine Design

(See Revs. 3973, 4145, 4188, 4217, 4227, 4537, 4555, 4561, 4562, 4564)

Fastening and Joining Methods

(See Rev. 4233)

Rheology

(See also Revs. 3956, 4378)

4256. Ericksen, J. L., Anisotropic fluids (in English), *Arch. Rational Mech. Anal.* **4,** 3, 231–237, Jan. 1960.

This paper modifies and improves earlier theories of the anisotropic fluid state, which is observed for certain substances at temperatures between those of the crystalline and the isotropic fluid state. In order to define the anisotropy, the present theory assumes in each fluid particle a single preferred direction, described by a vector n_j. In conjunction with the usual 5 variables of velocity components, density and temperature, the theory then deals with eight unknowns. For the determination of these unknowns one has, as in all theories of continuous media, the equations expressing the conservation of mass, linear momentum, mo-

ment of momentum and energy. In addition, properly invariant constitutive equations in sufficient number and an equation of state are postulated so that a complete determination of the unknowns is possible. For $n_j = 0$ the equations of this theory reduce to those commonly used for isotropic fluids.

H. L. Oestreicher, USA

4257. Sahrma, S. K., Visco-elastic steady flow (in English), *ZAMM* **39**, 7/8, 313-322, July/Aug. 1959.

Using a modified form of the stress-strain relations for visco-elastic materials, some problems on steady flow have been solved in a closed form. An extra normal effect lacking in both the Newtonian and the non-Newtonian approaches has been found. The analysis is shown to have application in the study of gels formed by lyophillic solutions, an extreme illustration of which is table jelly. The results are found to be in good agreement with experiments.

From author's summary by D. S. Berry, England

4258. Whitelaw, J. H., Viscosity of steam at supercritical pressures, *J. Mech. Engng. Sci.* **2**, 4, 288-297, Dec. 1960.

A Rankine-type viscometer was used to obtain data on the kinematic viscosity of steam. Pressures ranged from 200 kg/cm^2 to 800 kg/cm^2 and temperatures from 370° C to 650° C. New data at high pressures are presented, in which range there is no counterpart for purposes of comparison. The reproducibility of the work seems to be better than average, and there is good reason to believe that the new data are of good reliability both in magnitude and trend.

G. A. Hawkins, USA

4259. Jones, J. R., Flow of a non-Newtonian liquid in a curved pipe, *Quart. J. Mech. Appl. Math.* **13**, 4, 428-443, Nov. 1960.

Reiner-Rivlin's stress-strain relation is used for determining the flow of incompressible non-Newtonian liquid through a curved pipe. Differential equations are simplified by neglecting terms multiplied by the second power of the ratio radius of pipe/radius of curvature. The simplified differential equations are solved by a method of successive approximations.

L. S. Rintel, Israel

4260. Genensky, S. M., A general theorem concerning the stability of a particular non-Newtonian fluid, *Quart. Appl. Math.* **18**, 3, 245-250, Oct. 1960.

Author examines the stability of laminar flow for an incompressible, non-Newtonian fluid for which the stress tensor can be expressed as: $T = -pI + \phi_1 A_1 + \phi_2 A_2$, where I is the identity tensor, A_1 and A_2 respectively essentially the velocity gradient and tensors, and p, ϕ_1, and ϕ_2 scalar constants. The resulting stability criterion requires the ratio of the fourth derivative to the second derivative of velocity with respect to a coordinate normal to the velocity to be everywhere different than ρ/ϕ_2, where ρ is the density.

S. Gratch, USA

4261. Bird, R. B., New variational principle for incompressible non-Newtonian flow, *Physics of Fluids* **3**, 4, 539-541, July/Aug. 1960.

The principle of minimum dissipation is frequently invoked in various contexts but the limits of its validity are not known. In the present note these limits are investigated as far as the steady flow of incompressible liquids is concerned. It is assumed that the stress tensor in the liquid is equal to the rate of strain tensor multiplied by a scalar coefficient of viscosity. This scalar may depend upon the rate of strain tensor but only by the way of the diagonal sum of the tensor square, itself a scalar. Neglecting inertia terms and considering steady flow it is shown that the equations of motion can be derived from a variational principle according to which the volume integral of a functional of the velocity should be a minimum. This principle is mathematically equivalent to the equations of motion but does not lend itself to any simple interpretation. However, for Newtonian and special instances of non-Newtonian liquids (formula of Oswald and de Waele) this principle becomes identical with the principle of minimum dissipation.

R. Eisenschitz, England

4262. Andres, U. Ts., Equilibrium and motion of spheres in a viscoplastic liquid, *Soviet Phys.-Doklady* **5**, 4, 723-726, Jan./Feb. 1961. (Translation of *Dokladt Akad. Nauk SSSR (N. S.)* **133**, 4, 777-780, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

In viscous liquids, and in particular in structureless suspensions, all bodies with densities differing from that of the liquid always rise or fall. In contrast to this behavior, in viscoplastic media with a limiting shear stress τ_0 there always exists a critical size d_0 for bodies of any density, such that if the body dimension is less than d_0, gravitational forces will not exceed the structural forces in the liquid, and the body will remain motionless. In technological processes using viscoplastic liquids (the enrichment of useful minerals in a heavy suspension, hole-drilling and the removal of rock particles by clay solutions), the critical diameter of equilibrium particles and the law of motion for particles in a liquid possessing plasticity are of very real interest.

From author's summary

4263. Bartenev, G. M., On irreversible flow of rubberlike polymers, *Soviet Phys.-Doklady* **5**, 4, 853-856, Jan./Feb. 1961. (Translation of *Dokladt Akad. Nauk SSSR (N. S.)* **133**, 1, 88-91, July 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

Hydraulics

(See also Revs. 4170, 4315, 4341, 4342, 4350, 4377, 4503, 4525, 4531)

4264. Rowe, P. N., and Henwood, G. A., Drag forces in a hydraulic model of a fluidised bed: Part 1, *Trans. Instn. Chem. Engrs.* **39**, 1, 43-54, 1961.

This paper describes experiments that were made to measure the drag on a sphere in a water stream and how this drag varied with the presence of neighboring spheres. The spheres were 1/2 in. and 5/8 in. in diameter and the flow conditions gave Reynolds numbers in the range 10 to 1000.

Increases of drag coefficient of almost two orders of magnitude are obtained when the sphere forms part of a regular packed array. Very large changes are associated with the surfaces of particle assemblies. Repulsive forces between particles have been found and also conditions that lead to a reduction in drag.

The results are used to explain some of the features of fluidized beds. They indicate the need to know more of the gas flow patterns before these results can be applied to bubble formation in gas-fluidized beds.

From authors' summary by P. Franke, Germany

4265. Ibad-Zade, Yu. A., Velocity distribution across a channel (in Russian), *Gidrotekh. Stroit.* **30**, 7, 43-46, July 1960.

Average velocities along verticals of a stream are computed as exponential functions of the depth of the vertical, and a curve of distribution of such average velocities is drawn across the stream, which is used for a computation of water discharge by the Harlacher method. Author applies the exponential formula by Pavlovskii, and this is the only difference from the method by A. van Rinsum ["Abfluss in offenen natürlichen Wasserläufen," 1935], where simple Chézy formula was applied.

S. Kolupaila, USA

4266. Sdobnikov, D. V., Passage of ice and debris through spillways of the Lenin hydroelectric plant on the Volga (in Russian), *Gidrotekh. Stroit.* **31**, 1, 23–26, Jan. 1961.

A large dam was built across the Volga above Kuibyshev in 1955 to 1959. Passage of ice blocks, 2 ft. thick, presented no problem. However, very difficult was the elimination of an enormous amount of debris transported by water after flooding of the area of 2500 square miles. Trash from abandoned settlements, tree trunks, brushes and other litter clogged the entrance to the power plant and had to be carried out by cranes and boats. A trash gate proved to be unsatisfactory. Important conclusions are drawn.

S. Kolupaila, USA

4267. Reinius, E., The stability of the downstream part of earth dams, Trans. Roy. Inst. Tech., Stockholm no. 168 (Civ. Engng. no. 4), 44 pp., 1961.

Model tests on triangular sand fills are described, and the stability of the fills and the distribution of the earth pressure under the fills studied. Comparisons of the laboratory results and different methods of stability calculations are reported.

From author's summary

4268. De Ricco, G., Uniform motion in large pipes under pressure; application of the Colebrook formula (in Italian), *G. Gen. Civ.* **99**, 1, 29–36, Jan. 1961.

In the light of Colebrook formula author elaborates the results of experimental determinations of friction losses carried out in 1930-1935 by an Italian Commission on 48 large industrial pipes. From the experimental values of Re and λ, those of the relative roughness ε/D are deduced by means of Colebrook formula; for every pipe these should remain constant, when the discharge, and therefore Re, is changing. Experimental points in the graph $\varepsilon/D = f(Re)$ are interpolated linearly, using least squares method; the angular coefficient of the interpolating line is recognized very small and practically negligible for almost all pipes, and so the validity of the Colebrook formula is confirmed. From the calculations a large series of ε values is obtained. D. Citrini, Italy

4269. Jung, R., Calculation and application of ejectors (in German), *Forsch. Geb. Ing.-Wes.* (*B*) 26, 32 pp., 1960.

Momentum balance theory for ejectors with cylindrical mixing tube and conical diffuser. Incompressible flow only is considered but difference in density between driving and driven flow is taken into account. Losses due to mixing-tube wall friction and diffusor efficiency are combined into a single loss coefficient. Dependence of this coefficient on various parameters is studied experimentally. The resulting data permit design of ejectors of optimal efficiency for a given purpose. L. J. F. Broer, Holland

4270. Leith, W. C., and Thompson, A. L., Some corrosion effects in accelerated cavitation damage, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 795–807, Dec. 1960.

Authors first give a concise survey of research work done in the field of cavitation damage and then present additional experiments carried out on magnetostriction apparatus according to ASME standard procedure. Some effects of metal properties, liquid characteristics and influence of cathodic protection and chemical additives were investigated. The relationship of corrosion and erosion is discussed. Emphasis is laid on simulating actual service conditions in cooling system of diesel engines regarding the pressure and temperature and a good agreement with field experience is stated.

The discussion is a very valuable part of the paper and presents explanation of some details that was not given by authors.

M. Nechleba, Czechoslovakia

Incompressible Flow

(See also Revs. 3992, 4170, 4260, 4261, 4310, 4316, 4324, 4335, 4350, 4377, 4378, 4389, 4423, 4426, 4436, 4438, 4451, 4455, 4462, 4543, 4560)

4271. Apelt, C. J., The steady flow of a viscous fluid past a circular cylinder at Reynolds numbers 40 and 44, Aero. Res. Counc. Lond. Rep. Mem. 3175, 28 pp., 1961.

Paper describes the numerical solution of the Navier-Stokes equations by hand computation; apparently too much use is made of special devices requiring judgment for the use of an electronic computer.

The Reynolds number range is of interest because the transition between a steady flow and an unsteady vortex street is observed. Also there were certain discrepancies in the previous numerical solutions. Reviewer notes that the work of Imai [University of Maryland Technical Note BN-104] showing that the time average of the turbulent flow around a cylinder at a Reynolds number of 6000 closely resembled the flow at a Reynolds number of 40 adds to the importance of this Reynolds number range.

On the basis of the present calculations it is concluded that a numerical solution in agreement with experiment is obtained at Reynolds number 40 and it appears that the numerical procedure will converge at higher Reynolds number even though the steady solution may not be physically meaningful.

W. Squire, USA

4272. Diprima, R. C., The stability of a viscous fluid between rotating cylinders with an axial flow, *J. Fluid Mech.* **9**, 4, 621–631, Dec. 1960.

The stability characteristics of viscous flow between concentric rotating cylinders with an axial flow are of interest in several technical areas including paper-making and the design of rotating electrical machinery. A theoretical attempt of Goldstein (1937) and experimental results of Cornish(1933), Fage (1938) and Kaye & Elgar (1957) disagree with one another. In the present paper a new attempt has been made for a theoretical analysis with the assumption that the spacing between the cylinders is small compared to the mean radius and for small axial flow. The problem is treated either replacing the axial velocity by its average value or using a parabolic axial velocity distribution. It is found in both cases that the critical Taylor number increases rapidly from the correct value ($T \sim 1710$) at $R = 0$ to a value of about 7000 at $R = 60$ in qualitative agreement with the work of Kaye & Elgar.

W. Wuest, Germany

4273. Nigam, L. N., Shear flow past a wing of circular plan form, *J. Roy. Aero. Soc.* **64**, 599, 694–697 (Tech. Notes), Nov. 1960.

The solutions for potential flow past a wing of circular planform have been given in terms of spheroidal coordinates using the concept of acceleration potential. The solutions can be easily adopted to the case when the undisturbed flow is weakly sheared. In the particular case when the undisturbed velocity $U(z) = Az + V$, where A and V are constants, the pressure equation can be simplified by a simple substitution and the solution for a thin wing of circular planform can be easily constructed from the solution for potential flow. If the wings in the two cases develop the same lift, it is found that the wing in shear flow should be slightly cambered (potential wing is uncambered).

From author's summary by S. Otsuka, Japan

4274. Noce, R. E., and Daily, J. W., Roughness effects on frictional resistance of enclosed rotating disks, *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 553–562, Sept. 1960.

Already published research on fluid friction on enclosed rotating disks [AMR **14**(1961), Rev. 3091] is here extended to deal with

surface roughness effects. Experiments have been carried out with disk Reynolds numbers in the range 4×10^3 to 6×10^5 for three values each of the housing axial clearance and the roughness. Roughness was obtained by cementing commercial grit paper to the disk and housing surfaces: mean grit particle diameter was used as the characteristic roughness height, and ratios of roughness height/disk diameter were 1,000, 2,000 and 3,200. The test fluids were water and three lubricating oils and no through circulation of fluid was allowed during torque measurement runs.

In the earlier paper, the existence of four flow regimes, namely laminar or turbulent flows and merged or separate boundary layers on disk and housing, was demonstrated. The effect of roughness in the laminar regimes is now shown to be small, as would be expected. With turbulent flow, found to occur at Reynolds numbers above 2×10^5, the torque coefficients for rough disks rise above the smooth disk curve in a transition region. When the roughness effect is fully developed, the torque coefficient levels off to a constant value. There is no apparent difference for the cases of merged and separate turbulent boundary layers, and the effect of roughness outweighs the effect of housing clearance. Empirical expressions are given relating Reynolds number, torque coefficient and clearance and roughness ratios for the transition region.

In the discussion, the question of designing for miniumm disk friction is considered. The problem of disk design for maximum friction is also of interest in the dynamometer application, and reviewer suggests extension of measurements to effectively rougher surfaces, e.g. disks with holes or with surface projections.

J. A. Cole, Australia

4275. Power, G., and Jackson, H. L. W., Use of Stokes' stream function for discontinuities of potential at a spherical boundary, *Appl. Scient. Res. (B)* **8,** 5/6, 463–466, 1960.

A use of Stokes's stream function is presented which allows for discontinuities of potential at a spherical surface of separation. It is of special importance in heat problems where the radiation boundary condition applies. Previous results depending upon continuity of potential across the surface can be deduced.

From authors' summary by J. N. Hunt, England

4276. Frankl, F. I., Approximate calculation of the potential flow of a fluid jet flowing in a thin layer over the surface of a solid body, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **24,** 2, 546–548, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author exposes general expressions, in orthogonal curvilinear coordinates, for the motion and for the pressures of an ideal fluid flowing along an arbitrarily shaped surface, taking the thickness of the layer, in the direction of the normal to the surface, as a small quantity.

H. J. Schoemaker, Holland

4277. Orudzhaliev, E. A., The isothermal flow of a real gas at high pressure (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Neft' i Gaz.* no. 5, 115–122, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 3707.

Integration of the equation of motion of a gas in a tube, in the presence of frictional forces, furnishes analytical formulas for high-pressure gas mains with isothermal flow. The state equation is that suggested by K. V. Pokrovskii ["Thermodynamics of gases in the near-perfect state," Thesis, Akad. Nauk Azerb. SSR, Energetich. In-ta, 1946]. In the region of Reynolds numbers of $R > 4 \times 10^6$, the drag coefficient is determined by Nikuradse's relationship, while for $R < 4 \times 10^6$, E. I. Khodanovich's relationship (in: "Problems of the winning, conveying and processing of natural gases," Goshekhizdat, 1951), is used.

V. A. Bashkin

Courtesy Referativnyi Zhurnal, USSR

4278. Graebel, W. P., The stability of a stratified flow, *J. Fluid Mech.* **8,** 3, 321–336, July 1960.

This paper deals theoretically with the problem of the hydrodynamic stability of a stratified flow of a viscous fluid. The primary flow consists of two laminar streams of viscous fluids of different densities flowing in opposite directions between two parallel inclined planes under the action of gravity. The effect of surface tension at the interface of the two fluids is included in the formulation of the problem.

Since instability can be expected to occur at low Reynolds numbers when the inclination is nearly vertical, the solution of the Orr-Sommerfeld equations is developed as a power series in the transverse space coordinate. It is shown that, for the vertical case, the flow is unstable for all values of the Reynolds number. Surface tension is found to influence both the direction and celerity of the disturbance. Results are also given for inclinations slightly away from the vertical, where small critical Reynolds numbers do exist.

From author's summary by I. Proudman, England

4279. Bukhman, S. V., and Chernov, A. P., Investigations on binary-phase free jets (in Russian), Issled. Fiz. Osnov. Rab. Prots. Topok i Pechey, Akad. Nauk KazSSR, Alma-Ata, 1957, 175–189; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4031.

The first part discusses the phenomena of breakdown of droplets of different liquids (water, ethyl alcohol, glycerine, toluene) in air. The experiments on the disintegration of droplets were by spark photography of freely-falling droplets of the different liquids, while breaking up in a stream of air. The mechanism of droplet disintegration has been investigated. It has become evident that in determining the numerical value of the disintegration constant, attention must be paid to the turbulence characteristics of the stream (jet) in which the droplets are broken up (atomized). It is demonstrated that for a laminar flow the value of the disintegration constant (coefficient of atomization) is approximately 3.5. Experimental proof is given that the value of the coefficient of atomization does not depend on the velocity of the droplet, but rapidly decreases with increasing degree of turbulence of the flow. The second part of the paper presents the results of experimental investigations on the motion of solid particles in a free jet. A method of calculating the velocities of such solid particles is presented. It is shown that in dust-laden, free-air jets, the velocity of the particles of the solid phase, varies considerably from the velocity of the air stream—even with particles of relatively small dimensions, of the order of 50–70 μ. The relative velocity of the particles in the jet is proportional to their size and initial velocity and inversely proportional to the density and viscosity of the gaseous medium in the jet, the radius of the nozzle from which the jet issues, and the form coefficient of the particles. It is made evident that the ratio of the particle velocity to the air velocity in corresponding parts of the starting length of the jet is approximately a constant. Authors are of the opinion that the rotation or spinning of the particles in the jet is essentially due to collisions and rolling on the walls of the nozzle, as well as the velocity gradients across the air flow, and the irregular shape of the particles. It has been shown experimentally that the drag coefficient of particles of irregular shape is higher than the drag coefficient of a sphere. An empirical relationship is put forward for determining the drag coefficient of a particle of irregular form by means of the drag coefficient for a sphere.

Yu. A. Lashkov

Courtesy Referativnyi Zhurnal, USSR

4280. Sawyer, R. A., The flow due to a two-dimensional jet issuing parallel to a flat plate, *J. Fluid Mech.* **9,** 4, 543–560, Dec. 1960.

A semi-empirical and experimental analysis of the two-dimensional turbulent jet discharging parallel to a flat plate at some transverse distance from the surface of the plate. The entrainment

of fluid near the plate causes a pressure difference across the jet, curving the jet toward the plate. The jet divides on striking the plate, a portion of the volume flow being reversed into the cavity spreading from the jet nozzle to the flow attachment stagnation point.

The velocity profile in the pt is found to be similar to that in the two-dimensional plane jet and consequently Goertler's solution

$$\frac{u}{U_{max}} = \operatorname{sech}^2 \eta$$

is used, where $\eta = \sigma(y/x)$ is the dimensionless distance across the jet. The spread parameter σ, however, was found to differ from the plane jet case and was estimated to be equal to 15 for various geometrical and dynamical configurations.

Static pressure, velocity-moments, and cavity lengths are shown as a function of transverse location of the jet-slot.

S. Eskinazi, USA

4281. Segel, L. A., A uniformly-valid asymptotic expansion of the solution to an unsteady boundary-layer problem, *J. Math. Phys.* **39,** 3, 189–197, Oct. 1960.

A cylindrical body oscillates perpendicularly to its axis in a viscous incompressible fluid. For calculating the laminar flow round the body the Navier-Stokes equations are linearized by omitting the nonlinear convective terms. The linearized equations can be solved exactly for a circular cylinder; the solution has first been given by Stokes. Specializing to the case of high Reynolds numbers one gets a discrepancy between this solution and the corresponding one which is given by linearized boundary-layer theory. The reason for this discrepancy is discussed. In a qualitative way it is due to the neglect of the effect of the boundary layer on the external flow (displacement effect). Author outlines a method for solving the linearized Navier-Stokes equations, also for the case of a general cylindrical body by developing the solution in power series of a suitably chosen reciprocal Reynolds number. This method avoids the errors of boundary-layer theory.

E. Becker, Germany

Compressible Flow (Continuum and Noncontinuum Flow)

(See also Revs. 4121, 4256, 4319, 4323, 4326, 4334, 4402, 4460, 4489, 4521, 4542, 4562)

4282. Humphreys, J. S., On a circular cylinder in a steady wind at transition Reynolds numbers, *J. Fluid Mech.* **9,** 4, 603–612, Dec. 1960.

Some results of an experimental investigation of forces associated with the subsonic flow of air around a circular cylinder in a wind tunnel are presented. The oscillating forces due to the downstream vortex street are studied for Reynolds numbers in the "critical" range 4×10^4 to 6×10^5. Of particular interest is the observation, at the onset of transition to turbulence, of a spanwise wave or cell pattern near the cylinder surface, which is stabilized in a striking manner by the use of fine threads as a visualization technique.

From author's summary by J. Rom-Rabinowicz, Israel

4283. Liepmann, H. W., Gaskinetics and gasdynamics of orifice flow, *J. Fluid Mech.* **10,** 1, 65–79, Feb. 1961.

Paper gives the result of a study on the efflux of gases through circular apertures. The problem is considered as an example of a transition from the gas-dynamic to the gas-kinetic régime.

The mass flow of helium, argon and nitrogen was measured for a range of upstream pressures corresponding to (mean free path)/(aperture diameter) from about 50 to 5×10^{-3}; within this range the transition from molecular effusion to inviscid, transonic flow takes place. The theory for the two asymptotic limits is discussed and first-order corrections to the free molecular and inviscid limit formulas are given.

From author's summary by S. Uchida, USA

4284. Vinokur, M., Kinematic formulation of rotational gas flow, *J. Fluid Mech.* **9,** 4, 533–542, Dec. 1960.

It is shown that for the steady isoenergetic rotational flow of an ideal gas, both the specific enthalpy and the speed of sound can be expressed as functions of the velocity. As a result, it is possible to formulate the equations of motion so that the velocity is the only dependent variable. For a gas whose enthalpy and sound speed are functionally related, the results are a generalization of those for a perfect gas. If the enthalpy and sound speed are independent variables, the new formulation leads to a single vector equation whose solution completely determines the flow.

R. C. Binder, USA

4285. Smith, J. H. B., The properties of a thin conically cambered wing according to slender-body theory, Aero. Res. Counc. Lond. Rep. Mem. 3135, 23 pp., 1960.

Slender-body theory is used to calculate the lift and drag forces acting on a thin slender delta wing cambered to form part of the surface of a circular cone, in the type of flow in which separation is from the trailing edge only. The boundary condition satisfied by the flow on the wing surface is applied there, instead of on a nearby plane as is usual in linearized theory. This has relatively little effect on the over-all forces on a wing at a given incidence. However, a large discrepancy arises between the over-all forces at the incidence for which the singularity in the pressure at the leading edge vanishes, as calculated by the present and by the usual linearized theory. This is particularly important, since it is at this incidence that the type of flow treated is expected to be realized in a physical fluid. The lift-dependent drag factor found is below the usual linearized-theory value for this type of wing at the incidence of no leading-edge singularity; and, for large lift, is below unity, which is the minimum for a trailing vortex sheet which is effectively flat. From author's summary

4286. Ware, G. M., Low-subsonic-speed static stability of right-triangular-pyramid and half-cone lifting reentry configurations, NASA TN D-646, 28 pp., Feb. 1961.

An investigation has been made to determine the low-subsonic-speed static stability characteristics of several right-triangular-pyramid and half-cone configurations. Also studied were the effects of various modifications, such as base extensions, nose shape, nose incidence, and ridge-line shape. The investigation showed that, in general, the models had satisfactory longitudinal and lateral stability. The basic pyramid model and the conical ridge-line model with or without a rounded nose had almost identical longitudinal and lateral stability characteristics and lift-drag ratios. The lift-drag ratios of the cylindrical ridge-line and half-cone models were considerably lower than those of the conical ridge-line model. The addition of a 20° boattail to the models increased the lift-drag ratios but decreased the directional stability, whereas a streamwise base extension was more effective in increasing the lift-drag ratios and increased the directional stability.

From author's summary

4287. Neethling, J. D., On the non-existence of transonic perturbations, *Quart. Appl. Math.* **18,** 3, 229–233, Oct. 1960.

The method of derived characteristics developed by Schafer [AMR 7 (1954), Rev. 188; AMR 9 (1956), Rev. 3989] is used to

construct plane transonic potential flows past profiles. The author discusses flows past perturbations of basic profiles by superimposing additional terms on the basic solutions and shows that such terms are analytic functions of the hodograph variables. Conclusion is that singularities of curvature cannot exist and thus that the perturbations cannot be restricted to a finite segment of the supersonic boundary. This result is in agreement with the theorems of Morawetz [AMR 11 (1958), Revs. 3149, 3150], which cannot be applied directly to the flows discussed in this paper.

J. B. Helliwell, Scotland

4288. Rogers, E. W. E., and Hall, I. M., Wall interference at transonic speeds on a hemisphere-cylinder model, Aero. Res. Counc. Lond. Curr. Pap. 510, 26 pp. + figs., 1960.

Tests have been made in three National Physical Laboratory wind tunnels on a pressure-plotting model consisting of a long cylinder with a hemispherical nose. The surface pressure distributions were measured for stream Mach numbers between 0.7 and 1.1 at zero model incidence, and schlieren photographs were taken. The blockage ratios were 0.211%, 0.117% and 0.120%.

The principal feature of the flow is the effect of working section size on the rate at which the terminal shock wave moves back along the model with increasing stream Mach number. This is thought to depend mainly on the distance from the model to the slotted walls of the tunnel, and not necessarily on the blockage ratio. The distance of the solid sidewall is important in influencing the local Mach number ahead of the terminal shock, by reflecting the expansion-wave system originating near the model nose.

From authors' summary

4289. Henshall, B. D., and Cash, R. F., Observations of the flow over a two-dimensional 4 per cent thick aerofoil at transonic speeds, Aero. Res. Counc. Lond. Rep. Mem. 3180, 19 pp., 1961.

Flow photographs and detailed pressure distributions for a 4 per cent thick circular-arc biconvex airfoil at transonic speeds are presented. The results for incidences of 0, 1, 2 and 5 deg are analyzed in detail. From authors' summary

4290. Tirumalesa, D., and Satyanarayana, B., An experimental study of the effect of the plenum chamber size on the flow in a slotted wall transonic test section, *J. Aero. Soc. India* **12,** 3, 51–62, Aug. 1960.

This paper gives the results of an experimental investigation of the influence of plenum chamber size (i.e. depth and slotted plate length) on the zero-lift interference characteristics of a slotted wall transonic test section with a circular arc half model.

From authors' summary

4291. Cox, M., Static tests on a conical centerbody supersonic air intake with an auxiliary air inlet slot, Aero. Res. Counc. Lond. Curr. Pap. 515, 18 pp. + figs., 1960.

Tests on a sharp-lipped supersonic intake have shown that at zero flight speed the total pressure recovery is improved from 0.78 to 0.97 by opening an annular auxiliary air inlet, at a compressor entry Mach number of 0.5. At the same time the ratio of peak to mean velocity at the compressor entry is reduced from 1.3 to 1.1

A simple method is given of calculating changes in pressure recovery with intake flow rate and bleed slot opening, which should be applicable to other intake and auxiliary inlet configurations. From author's summary

4292. Krasil'nikov, Yu. I., Rolling moment due to sideslip for plane wings in supersonic gas flow (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **29,** 124–135, 1960.

By applying a neat formula which gives the pressure difference on a flat wing in terms of a certain contour integral along the supersonic leading edge (cited reference: E. A. Krasilshchikova, 1952), author calculates the rolling moment due to sideslip for rectangular, triangular, and pentagonal wings. Results are presented in explicit formulas made possible by replacing within certain of the regions (e.g. region influenced by apex of the triangular wing) the exact pressure distribution by an approximation so as to facilitate the integration. A limited numerical application illustrates the results. The lettering on Figs. 4 and 7 seems to be erroneous. A. Von Baranoff, Germany

4293. Buongiorno, C., and Ponzi, U., An experimental investigation on the interference effect of an annular wing on the aerodynamic characteristic of axisymmetric bodies at supersonic speeds (in Italian), *Aerotecnica* **39,** 6, 302–307, Dec. 1959.

An experimental investigation of a configuration consisting of a central body with large cone angle and cowling ring is presented. The purpose of this investigation has been to find the interaction effects of the cowling ring on the aerodynamic characteristics of the central body. Tests at an angle of attack from 4° to -4° and at a Mach number of 3.14 both on the central body and on the central body-cowling ring have been carried out. The results show that a decrease in drag produced by the cowling ring and published in an earlier work is present also at an angle of attack. The other important result is that the cowling ring interfering with the central body produces also a remarkable lift generation, which yields, together with the drag decrease, an increase of the ratio (C_L/C_D) from the value 1 of the central body alone, to the value 5 relative to the configuration central body cowling ring.

From authors' summary

4294. Gonor, A. L., Location of frontal wave in asymmetrical flow of gas at high supersonic speed over a pointed body, *ARS J.* **30,** 9, 841–842 (Russian Suppl.), Sept. 1960.

4295. Portnoy, H., The quasi-cylinder of specified thickness and shell loading in supersonic flow, *Aero. Quart.* **11,** 4, 387–395, Nov. 1960.

Paper treats, by operational calculus, linearized supersonic flow in and about a ducted body whose surface is almost cylindrical. Mathematical formalism is rather elegant. Theory is applied to axisymmetric "quasi-cylinder" with a slender conical central body. K. G. Guderley, USA

4296. Naylor, D., A non-linear boundary value problem in hypersonic gas dynamics, *J. Math. Mech.* **9,** 5, 665–680, Sept. 1960.

Paper considers plane isothermal ($\gamma = 1$) hypersonic flow past a finite wedge followed by an infinitely long boundary parallel to the undisturbed flow. The rarefaction waves originating at the shoulder interact with the attached bow wave and cause it to curve. The problem is transformed to a modified hodograph plane where it is linear and reduced there by a Riemann-Green formula to an integral equation which is solved approximately; the shock shape is found and the flow properties may then be computed.

L. Trilling, USA

4297. Anderson, A. B. C., Calculation of hypersonic shock-layer flow parameters, knowing shock-layer ratio of specific heats, *J. Aerospace Sci.* **27,** 11, 873–874 (Readers' Forum), Nov. 1960.

4298. Gadd, G. E., The possibility of normal shock waves on a body with convex surfaces in inviscid transonic flow (in English), *ZAMP* **11,** 1, 51–58, 1960.

In inviscid transonic flow past a body with a convex surface, shock waves normal to the surface cannot occur if the Mach number upstream of the shock is close to unity and the flow is free from singularities. In present work it is assumed a singularity exists

immediately downstream of shock; further, this singularity is assumed to be similar to that occurring in low-speed flow over a surface with a discontinuity in curvature.

Author shows argument that normal shocks cannot occur on convex surface is inconclusive if the singularity exists. Analysis does not prove normal shock can occur; however it demonstrates that a downstream singularity is a necessary and sufficient condition for the occurrence of a normal shock and vice versa. Need for more elaborate study is pointed out.

R. A. A. Bryant, Australia

4299. Grebenshchikov, S. E., and Raizer, M. D., Skin effect and shock waves in an induction gas discharge, *Soviet Phys.—JETP* **11,** 6, 1201–1202, Dec. 1960. (Translation of *Zh. Eksp. Teor. Fiz., SSSR* **38,** 6, 1665–1667, June 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

A gas discharge induced at a frequency of 300 cps is investigated. The shock wave was found to detach itself from the current layer under the conditions of the experiment. The current distribution in the discharge chamber is determined by the nature of the skin effect for metallic conductors.

From authors' summary

4300. Guess, A. W., Density compression ratio across relativisctic-strong-shock waves, *Physics of Fluids* **3,** 5, 697–705, Sept./Oct. 1960.

4301. Liu, V. C., and Inger, G. R., On almost-free-molecule flow through an orifice, *J. Aerospace Sci.* **27,** 11, 875–876 (Readers' Forum), Nov. 1960.

4302. Bryant, R. A. A., Diabatic nozzle flow with constant small stage efficiency, *J. Roy. Aero. Soc.* **64,** 598, 632–635 (Tech. Notes), Oct. 1960.

This two-page technical note defines the local expansion efficiency in a nozzle as the ratio of enthalpy drop which actually occurs over an incremental length to the one which occurs for isentropic change of state. Assuming that this efficiency remains constant, relations between area, temperature, Mach number, etc., are developed for heat addition. It is stated that this simplified analysis is compatible with more rigorous method such as Shapiro's analysis of one-dimensional flow.

H. P. Eichenberger, USA

4303. Cherry, T. M., Some nozzle flows found by the hodograph method: Parts 1 and 2, *J. Austral. Math. Soc.* **1,** 1, 80–94, Aug. 1959; **1,** 3, 357–367, Aug. 1960.

The differential equations describing fluid flow are linear if the hodograph method is used. In Part 1, author discusses the superposition of several flows to achieve flows exhibiting certain symmetries and to design nozzle shapes for which the flow is everywhere analytic and ultimately uniform, as required e.g. in the test section of a supersonic wind tunnel. A numerical example is given, consisting of a flow previously treated by Emmons by finite-difference methods. Unfortunately the paper by Emmons is omitted from author's bibliography, rendering a comparison difficult.

In Part 2, author investigates the superposition of flows of the type $T - aR + bU$, where T describes a certain transonic flow, R a certain radial flow, and U (cf. Part 1) a certain flow which is ultimately uniform. The dependence of various types of singularities and the "relative length" (i.e. the ratio of length to the breadth of the test section) of the nozzle on the parameters a and b is discussed.

P. M. Treuenfels, USA

4304. Loshkov, A. I., Flow of gas with local constraints in subcritical and supercritical regimes (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **29,** 106–118, 1960.

Author examines the qualitative aspects of flow phenomena in compressible gases associated with sudden contractions within ducts, by relating them to classical results on jet contractions (Borda mouthpiece, orifices with sharp edges). He attempts as well a one-dimensional flow analysis and applies general momentum laws. The existence of sub- and supercritical regimes within contracted ducts with separated flow regions is made evident. The simple qualitative results are compared with schlieren observations. Paper aims at preliminary exploration of more complex cases encountered in turbines and other fluid machinery.

A. Von Baranoff, Germany

4305. Erickson, W. D., Some real-gas flow parameters for air, *J. Aerospace Sci.* **27,** 9, 716–717 (Readers' Forum), Sept. 1960.

Boundary Layer

(See also Revs. 3981, 4260, 4264, 4274, 4281, 4282, 4313, 4384, 4405, 4407, 4427, 4430, 4454)

Book—4306. Meksyn, D., New methods in laminar boundary-layer theory, New York, Pergamon Press, 1961, xxiv + 294 pp. $12.

The object of this interesting work is to treat boundary-layer theory as a branch of applied mathematics utilizing well-known asymptotic techniques. The author has utilized such methods in a number of papers, which the reviewer must admit he found somewhat difficult to follow. The connected account given in this book is much easier reading.

Three classes of physical problems are considered: the incompressible boundary layer, natural and forced convective heat transfer and hydrodynamic stability. Background material includes chapters on mathematical techniques used (steepest descents, Euler's transformation, etc.) and on viscous flow theory. An account is also given of the usual methods of solving boundary-layer problem.

This book appears unique (in English at least) in offering a compact account of some very important mathematical methods and of some basic fluid mechanical problems. The price, however, seems rather high, particularly when compared with a treatise such as Schlichting's "Boundary layer theory" which contains much more material.

W. Squire, USA

4307. Dutton, R. A., The effects of distributed suction on the development of turbulent boundary layers, Aero. Res. Counc. Lond. Rep. Mem. 3155, 28 pp., 1960.

Experiments have been performed on the turbulent boundary layer with uniform suction and with zero pressure gradient. The test surface consisted of a uniformly perforated sheet which replaced the floor of a wind tunnel, the tunnel boundary layer being removed through a slot a short distance ahead of the test section. For several different suction velocities, and for different entry conditions, boundary-layer measurements were made on the perforated surface and on a porous surface formed by covering the perforated sheet with calendered nylon fabric.

For certain conditions it was found that both the thickness and the velocity profile of the turbulent boundary layer remained constant over virtually the full length of the suction surface, thus establishing the existence of a turbulent asymptotic layer. At sufficiently high rates of suction it was found that an initially turbulent layer reverted to the laminar asymptotic form.

From author's summary

4308. Johnston, J. P., The turbulent boundary layer at a plane of symmetry in a three-dimensional flow, *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 622–628, Sept. 1960.

Author calculates and verifies experimentally the characteristics of the incompressible turbulent boundary layer for the flow over a flat plate through which projects, at right angles, a wall normal to the flow direction. It is shown that for treating the three-dimensional turbulent boundary layer in the vicinity of the main flow streamline of the impact point (plane of symmetry), the ordinary momentum integral technique may be applied. For small angles the velocity profile is skewed relative to the plane of symmetry and consequently the corresponding terms are included in the momentum equation. The boundary-layer traverses and the visualization at limiting wall streamlines are in agreement with author's computations as regards the separation line formed at a certain distance from the impact point. The main conclusion is that for such cases of three-dimensional boundary-layers at a plane of symmetry, the well-known semi-empirical methods (Tetervin-Doenhoff, Rotta, Squire and Young) may be employed.

S. Savulescu, Roumania

4309. Mercer, A. M., The growth of the thermal boundary layer at the inlet to a circular tube, *Appl. Scient. Res.* (*A*) **9**, 6, 450–456, 1960.

Author presents an analysis of the thermal entrance region for Poiseuille flow in a tube at constant wall temperature different from the entering fluid temperature. The classical treatment of this situation by Graetz utilizes separation of variables which reduces the energy equation to a Sturm-Liouville problem. The first eigenfunction gives the solution far from the entrance of the tube and an increasing number of eigenfunctions are required to obtain accurate temperature distributions as the distance from the entrance is decreased. The present treatment is directed to the region close to the entrance and utilizes a boundary-layer-type transformation in the energy equation. The transformed equation is solved by a power series expansion and the results can be joined to the Graetz solution a short distance down the tube.

R. Seigel, USA

4310. Botchov, R., Simplified analysis of boundary-layer oscillations, *J. Ship Res.* **4**, 2, 37–54, Nov. 1960.

The Orr-Sommerfeld equation is treated approximately by concentrating, in turn, on inviscid flow, viscous effects at the wall, and viscous effects at the critical layer. Qualitative agreement is obtained with more precise analyses of neutral disturbances in flows of a boundary-layer type. Unstable boundary layers, nonlinear effects, and the effects of an elastic wall are briefly considered.

Author's simplified analysis is intended to clarify the essential processes, but reviewer believes that only a little clarification has resulted. The key destabilizing role of viscosity and most other features are explained more clearly and accurately in Lin's well-known book ["The theory of hydrodynamic stability," Cambridge University Press, 1955; AMR **9** (1956), Rev. 1491], particularly Chapter IV on "General theory". L. A. Segel, USA

4311. Hassan, H. A., On unsteady laminar boundary layers, *J. Fluid Mech.* **9**, 2, 300–304, Oct. 1960.

Author shows how the equation for the stream function of a two-dimensional laminar incompressible unstationary boundary layer can be reduced from three to two independent variables by a suitable transformation of these variables, provided the velocity of the outer flow depends in a special way on space and time coordinate. A second transformation renders the boundary conditions independent of the free-stream velocity and allows the expression of the solution in terms of universal functions by suitably developing in power series with respect to the transformed wall-parallel coordinate. As an example an expression for skin-friction in two special cases is given, which shows well-known properties of skin-friction in accelerated and in oscillating flow without resort to approximate methods.

Reviewer remarks that an important special case, not mentioned by author, are the boundary layers caused by shock, or expansion-waves (although compressible) which have been treated by similar methods (e.g. by Cohen, AMR **10** (1957), Rev. 4105; a summary has been given by reviewer, AMR **14** (1961), Rev. 882 and "Progress in aeronautical sciences" I, 104–173, 1961).

E. Becker, Germany

4312. Parr, O., Flow around an axially advancing rotating body of revolution (in German), *Jahrbuch Schiffbautech. Gesellsch.* **53**, 260–271, 1959.

The development of the boundary layer on an axisymmetrical body rotating around its axis has been measured. The free-stream velocity U (parallel to the axis) has been kept at a Reynolds number of $5 \cdot 10^5$ based on the radius; the rotational velocity (V) of the body was varied from 0 to 4 U. From the measured velocity profiles, the boundary-layer displacement thicknesses in axial and tangential direction have been derived and are plotted as a function of the axial length with V/U as a parameter. Similar plots are given for the momentum thicknesses, which are compared with calculations according to Truckenbrodt (1954) assuming either wholly laminar or wholly turbulent boundary layers. Agreement in the purely laminar region is good. But the measurements show a pronounced increase of the longitudinal thickness with increase in rotational speed which is not predicted by theory. The transition Reynolds number is plotted as a function of V/U. Finally the measured and calculated torque coefficients are compared. At high values of V/U there is a large discrepancy.

H. P. Eichenberger, USA

Turbulence

(See also Revs. 4282, 4308, 4439, 4440)

4313. Benjamin, T. B., Effects of a flexible boundary on hydrodynamic stability, *J. Fluid Mech.* **9**, 4, 513–532, Dec. 1960.

Purpose of paper is to examine theoretically the use of coatings of elastic materials to prevent transition from laminar to turbulent flow. Theory is extension to flexible boundary of the small-disturbance Tollmien-Schlichting stability theory and makes use of "Tietjens function" and other functions that occur in solution of Orr-Sommerfeld equation. It is shown how solutions for flexible wall can be obtained from solutions for rigid boundary.

Outline and discussion is given first for Tollmien-Schlichting stability theory for rigid wall, then for theory for flexible boundary. Theory is given both for a nondissipative and a dissipative flexible boundary. Behavior of flexible medium itself is also examined.

Practical requirements are discussed. For example, a conclusion is that to avoid Tollmien-Schlichting instability, the wave velocity of surface waves in absence of flow should coincide with Tollmien-Schlichting wave velocity at wavelength of "most dangerous" Tollmien-Schlichting waves. Moreover, damping should be large enough to prevent surface waves from developing but not so large that Tollmien-Schlichting waves are permissable. Author states that a boundary that is both soft and light, one whose elastic constants are of same order as the dynamic pressure of the flow, may be practical for use at high speeds. This surface should have a small damping to avoid Tollmien-Schlichting type of instability and a large enough wave speed without flow to avoid surface wave instability. Although paper is somewhat sketchy in places, it gives comprehensive coverage of stability of laminar flow over a flexible wall. N. Tetervin, USA

4314. Soo, S. L., Ihrig, H. K., Jr., and EL Kouh, A. F., Experimental determination of statistical properties of two-phase turbulent motion, *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 609–621, Sept. 1960.

The experimented two-phase flow consisted of solid particles (glass beads) conveyed by a turbulent air stream in a duct of 3-in.^{2} cross section. Stream velocities ranged from 25 to 85 fps; beads having 105 to 125 μ or 210 to 250 μ diameter were fed by a screw apparatus at flow rates up to 0.5 lb/min.

A tracer-diffusion technique for the determination of gas-phase turbulent motion was used instead of the well-known hot-wire technique, which was not possible because of the presence of fluid particles in the stream. Helium was employed as diffusion tracer and its concentration in the air was recorded at different stations through sampling probes. For the determination of motion of solid particles a high-speed photographic apparatus (from 5000 to 8000 exposures per second) was employed, giving successive images from which the particle diffusivity could be evaluated following a statistical scheme.

The results give the possibility to determine the relation between the turbulence characteristics of the two phases. The stream turbulence appears not to be significantly affected by the presence of the particles. The particle motion is nonisotropic, due to gravity and wall effects; its intensivity is greatly affected by the distribution of stream intensivity in the duct. The probability of particle-steam encounter has a significant effect on the particle diffusivity, resulting in the cases studied of the order of 0.01 of of the eddy diffusivity of the stream.

Reviewer feels that important contributions such as that given by authors should prove very fruitful also in the development of researches concerning sediment transport by water. Reciprocal knowledge on the two allied fields of research is desirable for future advancement. A. Ghetti, Italy

4315. Irmay, S., Accelerations and mean trajectories in turbulent channel flow, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 961–972, Dec. 1960.

Author shows that even in steady uniform turbulent channel flow there exist average mean accelerations and forces acting on fluid particles. These are computed from the flow equations and Laufer's empirical data [J. Laufer, NACA Reports 1053, (1951), and 1174, (1954)]. In this flow there exist also both normal and axial mean accelerations, the latter being highly negative (15 g and more) in the dissipative "transition" layer near the wall, and elsewhere constant. They occur in most river bed-load and dune sand movement. Mixing and sediment transportation are explained dynamically. Mean shear is shown to exist mainly in the dissipative layer. Mean turbulent trajectories are defined. The author's consideration of mean particle accelerations is a novel and informative approach to the consideration of turbulent flow in pipes and channels. In particular, it seems to have possibilities for the description of transport of sediment and suspended matter.

D. Gh. Ionescu, Roumania

4316. Polyakov, E. I., Experimental investigation of axially symmetric turbulent jets, *Soviet Phys. -Tech. Phys.* **5**, 10, 1173–1179, Apr. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30**, 10, 1238–1244, Oct. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

Paper presents the results of an experimental study of the principal axially symmetric forms of incompressible free turbulent jets. Simple equations are supplied for analyzing the principal parts of jets and are found to be in satisfactory agreement with experiment.

From author's summary

4317. Ozmidov, R. V., The part played by turbulent vortices of different orders of magnitude in the process of diffusion (in Russian), *Izv. Akad. Nauk SSSR, Ser. Geofiz.* no. 2, 272–273, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4004.

When studying turbulent diffusion on a water surface by means of "floating indicators", it has been found that small indicators diffuse more rapidly than those of larger size. The analysis, founded on the "two-thirds rule" of A. A. Kolmogorov and A. M. Obukhov, and on the assumption that the diffusion of a pair of indicators is solely influenced by turbulent vortices of orders of magnitude within the range (b, l), leads to the expression

$$\overline{\Delta^2_{b,l}}/\Delta^2_{0,l} = 1 - (b/l)^{2/3};$$

Wherein $\Delta\, b, l$ is the change (during a fixed time interval Δt) in the distance l between two indicators of magnitude b. This expression has been verified experimentally on scales of the order of meters (on the assumption that $\overline{\Delta^2_0, l} = 2k\,\Delta t \cdot l^{4/3}$); for a value of $k = 0.016$ cgs, good agreement with the experimental points was obtained. It is concluded that, in the orders of magnitude investigated, turbulence may be regarded as isotropic. A. S. Monin

Courtesy Referativnyi Zhurnal, USSR

Aerodynamics

(See also Revs. 4273, 4280, 4284, 4292, 4295, 4313, 4414, 4420, 4485)

4318. Fung, Y. C., Fluctuating lift and drag acting on a cylinder in a flow at supercritical Reynolds numbers, *J. Aerospace Sci.* **27**, 11, 801–814, Nov. 1960.

The unsteady aerodynamic forces on a circular cylinder in incompressible cross flow were measured on a stationary cylinder and one externally driven in harmonic oscillation across the wind stream. The model was 12.65 inches in diameter and 6 feet long. The center 22 inches consisted of a magnesium shell mounted elastically to the outboard part through strain-g[illegible] transducers. Forces acting on this center section were measured.

Tests were conducted through the Reynolds number (Re) range 0.2×10^6 to 1.4×10^6 (based on cylinder diameter). The model in forced harmonic oscillation was driven at amplitudes between 1/16 and 1/2 in. The lift and drag forces were nonharmonic and it was necessary to determine the statistical characteristics of these forces. Root-mean-square forces were obtained directly during testing, and peak values and power spectra were obtained from magnetic tape records of the strain-gage transducer output.

A most important feature of the observed forces was their randomness. On the stationary cylinder, peak lift and root-mean-square lift were constant at the high Re and increased rapidly as Re decreased toward critical. Power spectra for the lift appeared insensitive to Re. Peak values occurred at Strouhal frequencies considerably below the 0.18 to 0.20 range consistently observed by other investigators for the discrete frequency lift forces in the subcritical Re regime. Unsteady drag was much smaller than the lift. Mean drag was constant at all test Re and root-mean-square drag was constant at the high Re, again increasing rapidly as Re decreased toward critical. Limited drag spectra are given because inaccuracies in the data are large relative to the fluctuation magnitude. For the cylinder subjected to forced oscillations, no change in the lift and drag characteristics were discernible.

To the reviewer's knowledge the force measurements given in this report are the only ones that have been obtained for supercritical Re flow. D. R. Kobett, USA

4319. Krasil'shchikova, E. A., Wings at finite span with symmetric profile in subsonic and supersonic flow (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **27**, 29–37, 1960.

Paper represents a continuation and extension of author's earlier work [AMR 8(1955), Revs. 1073 and 1730, AMR 10(1957), Rev. 1548, AMR 11(1958), Revs. 4112 and 4191]. It is assumed that the wing is in nonuniform translatory motion in the direction of its axis of symmetry with given small additional symmetric oscillations of the upper and lower surface. Boundary conditions are derived for the disturbance velocity potential, which can be expressed in the form of a double integral, containing also the prescribed normal velocity components. In the case of uniform translatory motion separate expressions for the disturbance velocity potential in the subsonic and supersonic range are given and the integration intervals for both independent variables are fixed. If there is no oscillatory motion of surfaces, the expressions coincide with early results by Th. Karman and A. E. Puckett.

A. Kuhelj, Yugoslavia

4320. Pinsker, W. J. G., A semi-empirical method for estimating the rotary rolling moment derivatives of swept and slender wings, Aero. Res. Counc. Lond. Curr. Pap. 524, 17 pp. + figs., 1960.

A method is derived for estimating the derivatives l_p and l_r of swept and delta wings based on theoretical data and steady six-component wind-tunnel results. Good agreement is obtained with values of l_p measured on a rolling balance for a series of narrow delta wings. From author's summary

4321. Wells, E. W., Fatigue loadings in flight-loads in the nose undercarriage and wing of a Valiant, Aero. Res. Counc. Lond. Curr. Pap. 521, 15 pp. + figs., 1960.

Data obtained on the number of load cycles of various magnitudes occurring in the wing and the nose undercarriage of a Valiant in normal ground and flight conditions are presented. The conditions include taxiing, take-off, landing and flight in turbulence. An estimate is made of the loads in a typical operational flight to illustrate the relative importance of the various conditions.

A relationship is also established between the wing load and the acceleration at the aircraft c.g. when flying in turbulence. This enables the results for the flight tests to be linked to operational data obtained on gusts. From author's summary

4322. Kovaleva, V. A., On the nonstationary motion of a wing with rectangular planform, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 6, 1476–1491, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author applies operational techniques to obtain solutions of the linearized differential equation governing the supersonic nonstationary flow about a rigid rectangular wing. Two problems are treated. In the first problem the author considers the case of flight through a gust with vertical velocity given by $e^{\alpha t}(-\infty < t < 0, \alpha > 0)$. An exact solution of the linearized equations is obtained for this problem. In the second problem the author considers the general case of flight through a gust with arbitrary variation of vertical velocity. He assumes that the gust shape can be represented by a sum of terms of the form $e^{\alpha t}$ and synthesizes the solution from the results of the first problem. Since convergence of the series representing the gust shape cannot be assured, the solution of the general problem must be regarded as a type of asymptotic solution. M. Epstein, USA

4323. Morris, J., An investigation of lifting effects on the intensity of sonic booms, *J. Roy. Aero. Soc.* **64,** 598, 610–616, Oct. 1960.

Dealing with the shock noise associated with supersonic aircraft, author extends the theory of Whitham for volume effects on noise intensity by introducing the effect of lift variation. It is shown that the effect of lift is greater than had been expected and in fact dominates the shock noise above a critical height. For large aircraft (150,000 lb. A. U. W.) this noise level may well be objectionable at cruising altitudes. The majority of the calculations presented are for configurations with aft-located wings, for which it is assumed that the combined shock noise due to volume and lifting effects is equal to either the noise due to volume or the noise due to lift whichever is the greater. Based on this assumption charts are provided from which quick estimates of shock noise may be made for aft-wing combinations. This paper is a modified version of the author's paper bearing the same title and published as Boeing Report D6-3653, June 1959.

G. A. Tokaty, and D. J. Huggett, England

4324. Simpson, R. W., An extension of Multhopp's lifting surface theory, Coll. Aero., Cranfield, Rep. 132, 35 pp. + charts and tables, May 1960.

Multhopp's linearized subsonic lifting theory considers thin, continuous wing in nonviscous flow to be replaced by discontinuity sheet formed from doublets so selected that Kutta-Joukowski condition is satisfied at trailing edge. Resulting equation expressing relation between local downwash and local loading density for lifting surface is solved by constructing linear combinations of independent loading distributions made to satisfy equation at certain pivotal points. Present paper follows same general method, but extends theory to include case of discontinuity in chordwise direction, such as exists with deflected flap or aileron. Calculated results do not agree especially well with experiment; author ascribes this to viscous effects, and suggests correction by empirical modification of flap angle used. Calculation also yields certain qualitative results which may be useful.

C. W. Smith, USA

4325. Doetsch, K. H., Explanation of poorly damped lateral oscillations during automatic approach with aileron steering, Aero. Res. Counc. Lond. Rep. Mem. 3159, 7 pp., 1960.

Poorly damped Dutch-roll oscillations were experienced during landing approaches on different aircraft when controlled by an autopilot employing the aileron-steering technique. This phenomenon is explained with the aid of the time-vector representation and remedies are discussed. From author's summary

4326. Briggs, B. R., The numerical calculation of flow past conical bodies supporting elliptic conical shock waves at finite angles of incidence, NASA TN D-340, 62 pp., Nov. 1960.

The inverse method, with the shock wave prescribed to be an elliptic cone at a finite angle of incidence, is applied to calculate numerically the supersonic perfect-gas flow past conical bodies not having axial symmetry. Two formulations of the problem are employed, one using a pair of stream functions and the other involving entropy and components of velocity. A number of solutions are presented, illustrating the numerical methods employed and showing the effects of moderate variation of the initial parameters.

From the author's summary by H. C. Levey, Australia

Vibration and Wave Motion in Fluids

(See also Revs. 3980, 4281, 4313, 4323, 4394, 4438, 4447, 4448, 4521, 4529, 4561)

4327. Williams, W. E., Waves on a sloping beach, *Proc. Camb. Phil. Soc.* **57,** 1, 160–165, Jan. 1961.

Author's method for certain boundary-value problems in diffraction theory [*Proc. Roy. Soc. Lond.* (A) **252,** 376–94, 1959] is ap-

plied to title problem. The method provides a contour integral which is simpler and more general than those of Peters and Roseau, including both long and short waves, and valid for all beach angles. New short wave solutions are obtained for certain combinations of the parameters. Not all singularities at the origin are found to be logarithmic, some are algebraic.

J. N. Hunt, England

4328. Ball, F. K., Finite tidal waves propagated without change of shape, *J. Fluid Mech.* **9,** 4, 506–512, Dec. 1960.

The nonlinear equations of motion, based on the assumption of hydrostatic pressure, are modified by the insertion of Coriolis terms. The external applied pressure is such that a state of uniform steady motion is possible. All solutions representing waves traveling at constant speed without change of form are investigated. (It is well known that there are no such solutions when the Coriolis terms are absent.) By considering the phase plane it is shown that certain solutions are periodic in space, and some of these are illustrated in the paper. F. Ursell, England

4329. Gershman, S. G., and Tuzhilkin, Yu. I., Measurement of the transverse correlation coefficient of a continuous sound signal in the sea, *Soviet Phys. -Acoustics* **6,** 3, 291–297, Jan./Mar. 1961. (Translation of *Akust. Zh., SSSR* **6,** 3, 292–298, July/Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

The transverse correlation of a continuous noise signal with a frequency of 7.5 ± 0.2 kc was measured experimentally. With distances between the wave front and sound source r up to 12 km and with the receivers spread out over the wave front at distances d up to 3 km, the correlation coefficient remained near unity, independently of r and d. The small averaging time of the instrumentation (0.1 sec) made it possible to observe fluctuations in the correlation coefficient of a surface-reflected signal with the unreflected signal. The mutual-correlation coefficient of signals not touching the surface fluctuates very little. From authors' summary

4330. Romanenko, E. V., Distortion of a finite-amplitude waveform propagated in a relaxing medium, *Soviet Phys.-Acoustics* **6,** 3, 375–381, Jan./Mar. 1961. (Translation of *Akust. Zh., SSSR* **6,** 3, 374–380, July/Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

The distortion in waveform of a finite-amplitude wave during propagation in a solution of $MnSO_4$ is studied experimentally. The concentration was measured from 0 to 1.0 mole/liter. The working frequency range was 1.08-3.4 Mc. The pressure amplitude in the wave at the surface of the radiator was about 10 atm. The spectral composition of the distorted wave and phase relations between harmonic components vary as a function of distance to the radiator. The phase shift between the first and second harmonics is practically independent of the wave amplitude (as long as the amplitude remains finite) and is just half what it would be in the case of independent propagation of two waves of small amplitude with frequencies equal to the frequencies of the first and second harmonics of the distorted wave. From author's summary

4331. De Does, J. Ch., Experimental determination of bending moments for three models of different fullness in regular waves, *Inter. Shipbldg. Prog.* **7,** 68, 139–160, Apr. 1960.

4332. Lara, J. M., and Schroeder, K. B., Two methods to compute water surface profiles, *Proc. Amer. Soc. Civ. Engrs.* **85,** *HY* 4 (*J. Hydr. Div.*), 79–94, Apr. 1959.

4333. Chu, W.-H., Free surface condition for sloshing resulting from pitching and some corrections, *ARS J.* **30,** 11, 1093–1094 (Tech. Notes), Nov. 1960.

4334. Skvortsov, Yu. V., Komel'kov, V. S., and Kuznetsov, N. M., Expansion of a spark channel in a liquid, *Soviet Phys.-Tech. Phys.* **5,** 10, 1100–1112, Apr. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 10, 1165–1177, Oct. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

An investigation has been made of the initial stages of expansion of the channel of an intense spark in water for currents up to $7.5 \cdot 10^3$ amp in the discharge circuit and a rate-of-rise of $2 \cdot 10^{11}$ amp/sec. The rate of expansion of the channel boundaries, the velocity of the shock wave, the voltage in the spark channel, and the discharge time have been measured. Estimates are given of the conductivity and current density in the discharge channel. A hydrodynamic calculation of the pressure field and the velocity behind the shock-wave front is presented.

From authors' summary

Fluid Machinery

(See also Revs. 4270, 4274, 4283, 4304, 4312, 4348, 4387, 4413) 4414, 4416, 4531)

4335. Putiata, V. I., A special class of theoretical profiles (in Ukrainian), *Nauk. Zap. Kiivs'k. In-ta* **16,** 16, 117–134, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 3657.

A class of profiles is investigated into which closed curves Q_1, on a plane ζ_1, become conformally transformed by applying the generalized Joukowsky function. These curves Q_1 are in turn obtained by transforming a circle Q of radius r in the ζ-plane, by applying the function

$$\nu_1 = \nu + \lambda F_1(\nu) - i\mu F_2(\nu);$$

$$\nu_1 = \ln \frac{\zeta_1}{qr} - i\pi; \qquad \nu = \ln \frac{\zeta}{r} - i(\pi - \beta);$$

$$F_1(\nu) = 1 + \sum_{n=1}^{n=p_1} a_n e^{-n\nu}; \qquad F_2(\nu) = 1 + \sum_{n=1}^{n=p_2} b_n e^{-n\nu}$$

Here, λ, μ are small real numbers, a_n, b_n real parameters; β angle between the material axis and the radius of the point of intersection of Q and Q_1, q a real number smaller than $\cos\beta$. The coefficients a_n, b_n are linked by the following relationship:

$$\sum_{n=1}^{n=p_1} a_n \cos n\pi \sum_{n=1}^{n=p_2} b_n \cos n\pi = -1$$

Expressions are derived for the geometrical characteristics of the profile and for determining the velocity field. Expressions are found for the lift, moment, and coordinate of the focus. For a three-parameter family of profiles approaching the Joukowsky forms, an approximate relationship is established between the geometrical characteristics and the velocity distribution.

G. G. Tamashev

Courtesy Referativnyi Zhurnal, USSR

4336. Polasek, J., Induced velocities in the two-dimensional cascade theory, *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 355–356 (Brief Notes), June 1960.

Book—4337. Mosonyi, E., Water power development, Vol. 2: High-head power plants, midget stations and pumped-storage schemes, Budapest, Publishing House of the Hungarian Academy of Sciences, 1960, 1139 pp. + diagrams.

English edition of the 1st volume, covering low-head power plants, was favorably evaluated in AMR **11** (1958), Rev. 4064. Credit again is due to the Hungarian Academy of Sciences for per-

fect edition of the 2nd volume. It contains mainly high-head power plants (types; free-flow conduit developments; pressure tunnels, penstocks and valves; power stations at surface and underground; location of the station at low and high dams; hydraulic machinery and electrical equipment). Other chapters deal with small power plants equipped with water wheels or turbines (these could better fit into 1st volume) and pumped-storage developments, a very important addition to power plants. Text is concise and skillfully illustrated. In spite of this author was unable to cover river barrages and valley dams with their structures and equipment; those topics were transferred to a 3rd volume. An English edition of that volume will be also welcomed. Broad information on power development the world around and an extensive bibliography contribute to the great value of this unusual treatise by the renowned Hungarian scientist.

S. Kolupaila, USA

4338. Rex, E. M., Controlled radial energy gradient in axial flow compressors, *Aero/Space Engng.* **19,** 10, 22–23, 64, Oct. 1960.

An approach to axial flow compressor design which permits the designer to use two-dimensional analysis for high-performance engines.

From author's summary

4339. Carter, A. D. S., Moss, C. E., Green, G. R., and Annear, G. G., The effect of Reynolds number on the performance of a single-stage compressor, Aero. Res. Counc. Lond. Rep. Mem. 3184, 26 pp., 1960.

The Reynolds-number effects on the performance of a single-stage compressor have been measured over the range 0.08×10^6 to 9.0×10^6. The tests were carried out in a variable-density return-circuit rig and the Reynolds number varied partly by fluid density and partly by speed.

The main conclusion arising from this work, and from an appreciation of other results, is that the critical Reynolds number for a compressor blade is approximately 0.5×10^6. This is about half the value previously assumed. Above the critical Reynolds number the efficiency variation can be expressed by the equation $(1 - \eta_{max}) = kR^{-0.2}$. In this region the fluid outlet angles and hence the stage temperature-rise coefficient remain constant.

Below the critical Reynolds number the efficiency can be expressed by $(1 - \eta_{max}) = kR^{-0.5}$. Some variation of the stage temperature-rise coefficient can occur in this region.

From authors' summary

4340. Crouse, J. E., Montgomery, J. C., and Soltis, R. F., Investigation of the performance of an axial-flow-pump stage designed by the blade-element theory-design and overall performance, NASA TN D-591, 33 pp., Feb. 1961.

4341. Danel, P., and Duport, J., The selection of length and head scales for cavitation tests, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 784–794, Dec. 1960.

Authors start from the fact that the vertical dimensions of fluid machinery affect the Thoma cavitation number and the zone of cavitation. They explain this fact and recommend for model cavitation tests of hydraulic turbines to scale down the head in the same manner as the dimension of the turbine, according to Froude's law. Difficulties connected with this are described and scale effects due to critical pressure, gaseous diffusion, vaporization pressure, geometrical discrepancies and fluid friction are outlined.

Although the discussion in general does not agree with the suggestion of the authors, in reviewer's opinion the influence must not be disregarded in very large turbines operating under very low heads. This problem was mentioned in Spannhake's book "Kreiselräder als Pumpen und Turbinen" (Centrifugal Pumps and Turbines), Berlin, Springer, 1931, pp. 175–177 and reviewer attemped a method for correction of the Thoma number in this direction in the article "Sur la fonction de cavitation" (On the function of cavitation) in *Houille Blanche* **2,** 2, 117–120, 1947.

M. Nechleba, Czechoslovakia

4342. Wood, G. M., Murphy, J. S., and Farquhar, J., An experimental study of cavitation in a mixed flow pump impeller, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 929–940, Dec. 1960.

The three tested impellers, testing arrangement and procedure are described in detail and results reproduced in graphs. The results are studied on the basis of parameters: net positive suction head, suction specific speed and cavitation index.

Most interesting is that the minimum value of cavitation index occurred at no incidence flow condition. To avoid incidence effects, the inlet vane angle must be adjusted for the vane blockage effect. A speed effect was noted in that cavitation index increased with a decrease of velocity at a constant flow coefficient, which is explained by longer time of the bubble travel in the cavitation zone. The break off of the performance always took place when the cavitation zone stretched over the half length of the impeller blades, as is derived from the tip static pressure distribution curves.

M. Nechleba, Czechoslovakia

4343. Shivers, J. P., Hovering characteristics of a rotor having an airfoil section designed for flying-crane type of helicopter, NASA TN D-742, 24 pp., Apr. 1961.

Results of an investigation, conducted on the Langley helicopter test tower, of a rotor having an NACA 63_3A015 airfoil thickness distribution in combination with an NACA 230 mean line are presented. Comparison with a previously reported test of a symmetrical rotor blade having similar thickness distribution indicates that the present rotor efficiency was substantially improved over a wide range of tip Mach numbers. The maximum mean lift coefficient was essentially unchanged from that obtained with uncambered blades. Some data showing the effect of a distributed type of leading-edge roughness are also included.

From author's summary

4344. Langdon, G. F., and Neale, M. C., An analysis of the lateral-directional stability and control of the single-rotor helicopter, Aero. Res. Counc. Lond. Rep. Mem. 3149, 28 pp., 1960.

The lateral-directional stability of a single-rotor helicopter is investigated by solving the equations of motion for a typical aircraft. It is found that the motion has three constituent parts, two subsidences and an oscillation. The oscillation is slightly unstable at very low speeds but becomes more and more damped as the speed of the aircraft is increased. The response to disturbances of various kinds and to control movements is illustrated by means of examples. It is found that increasing altitude reduces the damping of the oscillatory motion and decreases the rolling effectiveness of the control. Flight measurements of the lateral-directional oscillation agree well with theory except for an overestimation of period at very low speeds.

Consideration is given to the effect of varying some design parameters; in particular it is shown that a large increase in roll damping at very low speeds will stabilize the lateral-directional oscillation and that a decrease in roll damping at high speed can lead to the appearance of a second oscillatory mode which may be unstable.

An appendix shows the connection between certain stability derivatives and control positions in asymmetric flight.

From authors' summary

4345. Yaggy, P. F., and Goodson, K. W., Aerodynamics of a tilting ducted fan configuration, NASA TN D-785, 15 pp., Mar. 1961.

Results of tests of a wing-tip-mounted ducted fan for V/STOL airplanes indicate longitudinal trim and control problems. However, it is shown how these problems may be alleviated by the use of exit vanes in the duct and a variable-incidence horizontal stabilizer. An advantage of this concept is the ability to vary the thrust vector independently of the wing angle of attack, making it possible to keep the wing unstalled even in descending flight.

From authors' summary

4346. Wittenberg, H., An extension of Bennett's formula for the profile-drag power of helicopter rotors, *J. Roy. Aero. Soc.* **64,** 600, 764–765 (Tech. Notes), Dec. 1960.

4347. van Leeuwen, H. P., Reduction of bending moment at the root of a rotor blade—effects of replacing elastic beam-type flapping hinge by leaf springs, *Aircr. Engng.* **32,** 380, 309–310, Oct. 1960.

It is shown that a reduction of the bending moments in a helicopter rotor blade may be achieved by replacing an elastic beam-type flapping hinge by a stack of leaf-springs having the same over-all bending stiffness but no shear stiffness. This reduction becomes more pronounced when the tensile load on the flapping hinge is increased. From author's summary

Flow and Flight Test Techniques and Measurements

(See also Revs. 3981, 4264, 4343, 4398)

4348. Sherman, J., Grochowski, F. A., and Sharbaugh, J. E., Variable flow resistance with adjustable multihole orifice plates in series, *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 645–653, Sept. 1960.

Tests performed on multihole and multislot orifices in series showed that a considerable range of flow resistance could be obtained when the alignment of penetrations is varied. For the area ratio (0.221) tested which gave the largest resistance flow in the case of the multihole orifices, the results showed a range of pressure loss coefficient of about 1 to 6 when the holes are varied from the in-line to the maximum misaligned condition. The loss coefficients are based on the velocity through the orifices.

From authors' summary by E. Eujen, Germany

4349. Lohrenz, J., Swift, G. W., and Kurata, F., An experimentally verified theoretical study of the falling cylinder viscometer, *AIChE J.* **6,** 4, 547–550, Dec. 1960.

A theoretical analysis was made for laminar fluid flow in the annulus of a falling cylinder viscometer. A viscometer calibration constant was defined from the results of this analysis. This constant was expressed in terms of only the physical dimensions of the viscometer. The validity of the theory was demonstrated by the agreement between predicted and experimental values of the viscometer constant.

Methods of representing calibration data were compared. Temperature and pressure effects on the viscometer constants were related to the mechanical properties of the viscometer materials.

The results of this investigation show that the practical design of falling cylinder viscometers is possible.

From authors' summary by W. C. Griffith, USA

4350. Francis, J. R. D., A further note on the speed of floating bodies in a stream, *J. Fluid Mech.* **10,** 1, 48–50, Feb. 1961.

A tripartite treaty is announced between Professors Prandtl (Univ. of Göttingen), and Hellström (Roy. Inst. of Tech. of Stockholm) and the author (Imperial Coll. of London) pertaining to the relative velocity of floating bodies and of the upper layers of free surface streams. In his paper, AMR **11** (1958), Rev. 1325, author found the unexpected result that the afore-mentioned relative velocity is zero. Present paper describes carefully conducted experiments which extend the range of the size of the previous floats and also investigates the effect of the turbulent fluctuations responsible for the interchange of momentum between float and water.

Author finds that floats can only travel faster than the stream if they are large enough to modify the neighborhood flow. This requirement explains Hellström's finding that a log, large compared to the depth of water, travels faster than the stream. Author's experimental results also prove Prandtl's proposed explanation of the phenomenon. V. G. Szebehely, USA

4351. Vereshchagin, L. F., Galaktionov, V. A., Semerchan, A. A., and Slesarev, V. N., Apparatus for high pressure and high temperature with a conical piston, *Soviet Phys.–Doklady* **5,** 3, 602–604, Nov./Dec. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **132,** 5, 1059–1061, June 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4352. Haines, A. B., Some notes on the flow patterns observed over various swept-back wings at low Mach numbers (in the R. A. E. 10-ft. × 7-ft. high speed tunnel), Aero. Res. Counc. Lond. Rep. Mem. 3192, 22 pp., 1960.

This paper brings together the flow patterns observed by an oil film (titanium oxide) technique over ten different sweptback wings at high incidence and low Mach number in the Royal Aircraft Establishment 10-ft × 7-ft High-Speed Tunnel. The designs range from 0.04 to 0.10 in mean thickness/chord ratio and from 40 deg to 60 deg in angle of sweep. Almost all the wings suffer from a leading edge separation and the paper discusses in general terms how the regions of flow separation, together with their associated part-span vortex sheets, are affected by changes in incidence, Reynolds number and wing design. Some brief reference is made to how the flow patterns are related to the over-all force and moment characteristics and how these characteristics might be improved by the use of different types of modification.

It appears that some correlation may ultimately be established between the spanwise extent of the separation over a sweptback wing at a given incidence with concepts based on two-dimensional flow and the pressure distribution over the swept wing in potential flow. No correlation can be expected in terms of the chordwise extent of the separation, in view of the presence of "part-span vortex sheets". The need for research into the "bursting" of "short" separation bubbles in two-dimensional flow and into the nature and reasons for the part-span vortex sheets in three-dimensional flow is especially emphasized. From author's summary

4353. Igglesden, M. S., Wind tunnel measurements of the lift-dependent drag of thin conically cambered slender delta wings at Mach numbers 1.4 and 1.8, Aero. Res. Counc. Lond. Curr. Pap. 519, 22 pp. + figs., 1960.

Lift and drag have been measured, at Mach numbers 1.4 and 1.8, on two sets of thin slender delta wings (aspect ratio 4/3) with differing degrees of conical leading edge camber, one set having drooped edges, and the other having edges shaped to give parabolic upwash distributions over the cambered part. An uncambered wing was included. All had sharp leading edges.

The absence of a realistic thickness distribution is thought to have led to unfavorable pressure fields such that separation-free flow was never achieved, and to this is attributed the failure to realize the theoretical drag reductions at the design lift conditions.

Information on the effective leading-edge suction, the nature of nonlinearities in lift-curve slope and the influence of free and fixed boundary-layer transition on the chord force are presented and discussed. From author's summary

4354. Bateman, P. J., Wind tunnel measurements of lift, drag and pitching moment of two highly swept ($\Lambda_L \doteq$ 87° and 81°) delta wing-body combination models with small tip fins at M = 2.47, Aero. Res. Counc. Lond. Curr. Pap. 514, 12 pp. + figs., 1960.

Tests have been made to explore the aerodynamic possibility of a feasible aircraft design based on a highly swept (over 80°) delta planform with sufficient body depth in the region of the center of gravity to accommodate the lifting jet engines required for take-off and landing. Two models were tested, of 87° and 81° leading edge sweepback respectively.

The initial lift curve slope is greater than slender-body theory would predict for the wing-body combination, due partly to the presence of tip fins, and the subsequent nonlinearity is greater for the more highly swept model. Drag measurements agree with estimates reasonably well and full-scale lift/drag ratios of 4.2 for the 87° configuration and 5.8 for the 81° configuration are indicated on the basis of the tunnel results.

The pitching moment coefficient measured about the mean quarter-chord point is shown to vary fairly linearly with C_L for the 81° model, but the aerodynamic center tends to move forward slightly (i.e. destabilizing) with increasing incidence for the 87° model.

From author's summary

4355. Squire, W., A shock wave reactor for high temperature gas phase reactions, *Research, Lond.* **14,** 88-92, Mar. 1961.

The possibility of using a "continuous" shock tube, based on the Gatling gun principle, for the production of chemicals is discussed. Application of the device to NO production from air and to acetylene production from methane is considered in qualitative terms. Commercial production of chemicals in shock tubes has been considered by several independent groups of investigators, perhaps most seriously by J. L. Lauer of the Sun Oil Company.

S. S. Penner, USA

4356. Warren, W. R., Kaegi, E. M., and Geiger, R. E., Shock tunnel experimental techniques for force and moment and surface flow direction measurements, *ARS J.* **31,** 1, 82–83 (Tech. Notes), Jan. 1961.

4357. Powell, H. N., Shock tube design for producing high gas temperatures, *ARS J.* **30,** 10, 980-982 (Tech. Notes), Oct. 1960.

4358. Schowalter, W. R., and Blaker, G. E., On the behavior of impact tubes at low Reynolds numbers, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 136–137 (Brief Notes), Mar. 1961.

Thermodynamics

(See also Revs. 4156, 4226, 4258, 4284, 4397, 4399, 4414, 4422)

4359. Tribus, M., Information theory as the basis for thermostatics and thermodynamics, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 1–8, Mar. 1961.

Information theory is used as a basis to derive thermodynamics and statistical mechanics. Entropy is introduced first and after that thermodynamics and statistical mechanics. This paper is based upon the papers by E. T. Jaynes [*Phys. Rev.* **106,** p. 620, 1957; **108,** p. 171, 1957; See AMR **11** (1958), Rev. 2293]. It is claimed that this method makes it easier for a student to learn thermodynamics, and this may well be the case in this reviewer's opinion.

D. Ter Haar, England

4360. Rastogi, R. P., and Srivastava, R. C., Generalized theory of thermal transpiration and thermal diffusion based on the thermodynamics of irreversible processes, *Physica* **25,** 391–397, 1959.

This theory is developed for a mixture involving a single chemical reaction. The theory is valid even when a nonlinear relation between chemical reaction rate and affinity exists, but the domain of validity of the theory cannot exceed the range of validity of the Gibbs formula for entropy production.

From authors' summary by P. E. Kriezis, Greece

4361. Czaja, W., Application of thermodynamics of irreversible processes to conduction phenomena in semiconductors (in German), *Helv. Phys. Acta* **32,** 1, 1–23, 1959.

The principle of minimum entropy-production in a stationary state of a system is discussed under most general conditions. It is assumed to have an unequal number of even and odd forces with additional conditions between these forces. Some isothermal as well as nonisothermal effects are calculated: general equations governing the behavior of the isothermal p–n-junction are derived and the heat conductivity of a homogeneous semiconductor under various conditions is calculated. Further applications of the theory are discussed.

From author's summary by P. E. Kriezis, Greece

4362. Terletskii, Ya. P., The causality principle and the second law of thermodynamics, *Soviet Phys.-Doklady* **5,** 4, 782–785, Jan./Feb. 1961. (Translation of *Dokladt Akad. Nauk SSSR (N. S.)* **133,** 2, 329–332, July 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4363. Stupochenko, E. V., Stakhanov, I. P., Samuilov, E. V., Pleshanov, A. S., and Rozhdestvenskii, I. B., Thermodynamic properties of air in the temperature interval from 1000 to 12,000 K and the pressure intervals from 0.001 to 1000 atm., *ARS J.* **30,** 1, 98–112 (Russian Supplement), Jan. 1960. (Translation of Physical Gas dynamics, USSR Akad. Sci., 3–38).

Partition sums for O_2, N_2, NO, A molecules, (disassociated) atoms, and ions are calculated from spectroscopic data on rotational, vibrational and electronic states. In calculating the composition of the air, the n-th fold ionization is shown to occur largely when the $(n-1)$st ionization is completed. Deviation of the gas from ideal behavior due to coulomb interaction and the effects of degeneracy are shown to be negligible. The solution of equations of chemical equilibrium of ideal gas systems is found using numerical methods. It is suggested that the Newton-Rafson method is preferable to iteration for reasons of convergence. Authors give a thermodynamic analysis of reacting ideal gases, and contrast it with Epstein's scheme. Graphical results include as functions of temperature and at pressures of 10^{-3}, 1, 10^{3} atmospheres: molar fractions of molecular and atomic components of air and electrons, enthalpy, specific heat, molecular weight, entropy of air, and velocity of sound in air.

G. N. Hatsopoulos, USA

4364. Hansen, C. F., and Hodge, M. E., Constant entropy properties for an approximate model of equilibrium air, NASA TN D-352, 33 pp., Jan. 1961.

Approximate analytic solutions for properties of equilibrium air up to 15,000°K have been programmed for machine computation. Temperature, compressibility, enthalpy, specific heats, and speed of sound are tabulated as constant entropy functions of temperature. The reciprocal of acoustic impedance and its integral with respect to pressure are also given for the purpose of evaluating the Riemann constants for one-dimensional, isentropic flow.

From authors' summary

4365. Erdelyi, I., On effect of centrifugal field strength upon thermal condition of gases. Explanation of Ranque effect (in Hungarian), *Energia es Atomtechnika* **14,** 1, 1–7, Jan. 1961.

The heat condition of gases in a centrifugal field is examined. As shown by the calculations in such a field an acceleration rate of 10^9 g can be achieved. Such accelerations can lead to temperature gradients higher than 10°/mm. This effect can be used to explain the phenomenon taking place in Ranque's tube. On the basis of the effect described ways and possibilities are investigated to achieve higher temperature differences in the Ranque tube. It is stated that the temperature difference is independent of the measures of the tube and the pressure of the gas at the inlet; it is, however, dependent on factor κ on the ratio of steam pressure before and after the tube as well as on the absolute temperature of the gas at the inlet. From author's summary

4366. Carter, Enid, Thermodynamic charts for the decomposition products of 80, 85, and 90 per cent w/w hydrogen peroxide (H. T. P.), Aero. Res. Counc. Lond. Rep. Mem. 3158, 9 pp. + charts, 1960.

Thermodynamic charts showing enthalpy, entropy and specific volume for a range of pressures and temperatures have been constructed for the decomposition products of 80, 85, and 90 per cent w/w hydrogen peroxide (H. T. P.) for regions above and below the saturation line. Examples are given to show how gas velocity, venturi nozzle throat and exit areas, exit pressure, specific heat at constant pressure, γ, characteristic velocity and thrust coefficient can be deduced from the charts and the rider scale which accompanies the paper. From author's summary

4367. Kreutzer, K., The creation of temperatures in the region of absolute zero (in German), *ZVDI* **103**, 3, 89–97, Jan. 1961.

Modern methods to reach very low temperatures are based on the adiabatic evaporation of liquid helium which process allows to start from 1 °K for further degrees of temperature. Continuously working helium cryostats are available nowadays which maintain a temperature of 0.5 °K over very long periods. By applying the magnetic cooling of paramagnetic salt and the nuclear cooling in combination with such cryostats further advances were achieved down to the very low temperature of $1.5 \cdot 10^{-6}$ °K. Considerations on the physical effects as used for these methods to decrease the temperature (these effects being the Joule-Thompson effect, the super-fluidity, the super-conductivity, the magneto-caloric effect, the mechano-caloric effect) and on the experiences gained hitherto confirm the principle that absolute zero is unattainable.

From author's summary

4368. Remenyi, K., Problems of thermal feed water deairation (in Hungarian), *Energia es Atomtechnika* **14**, 1, 17–21, Jan. 1961.

Heat and Mass Transfer

(See also Revs. 3959, 3981, 3984, 4018, 4156, 4258, 4275, 4306, 4360, 4361, 4401, 4407, 4414, 4423, 4430, 4458, 4517, 4518)

4369. Goodman, T. R., The heat-balance integral—further considerations and refinements, *ASME Trans.* **83**C (*J. Heat Transfer*), 1, 83–86, Feb. 1961.

Paper contains an extension of the integral energy equation to conduction problems involving time-dependent temperature fields as well as variable properties. Useful transformations are given, but the results of the paper must be used with circumspection and with due attention being paid to the mathematical imperfections inherent in any integral schemes (so-called Pohlhausen methods). These have been clearly enumerated by K. T. Yang in his discussion of the paper. J. Kestin, USA

4370. Ansorge, R., The control of preheat in cylinders (in German), *Ing.-Arch.* **30**, 1, 24–41, Jan. 1961.

This is a very fine example of how much remains to be said about several important problems, the theoretical side of which is often considered as quite clear and generally known. The question is on a circular cylinder of finite length and with zero initial temperature, surrounded either by a time-independent or by a linearly varying temperature field. Among a number of exact results of an unusually careful mathematical discussion of the general solution special mention should be made of an elegant proof of the fact that, with the increasing length of the cylinder, the above solution of the three-dimensional problem converges uniformly, in regard to the time coordinate, to the formula giving the two-dimensional solution of the plane problem.

Nevertheless, the main value of the paper lies in a detailed numerical and graphical examination of the important question: in what measure the resulting temperature distribution in the cylinder is influenced by its finite length and by the magnitude of the surface conductance between the body and its surroundings. Carefully elaborated tables and instructive diagrams bring valuable information of when and with what degree of approximation one can calculate the temperature field in a given cylinder of finite length from the simpler formula holding for infinite cylinders.

Technical and physical importance of the paper is obvious. Reviewer gladly recommends it to interested specialists.

V. Vodicka, Czechoslovakia

4371. Wolff, T., The Dirac delta function in heat conduction theory (in German), *ZAMM* **40**, 9, 421–422, Sept. 1960.

Author considers problems of nonsteady heat conduction, with internal heat generation, in both infinite and finite bodies. He demonstrates simplicity and ease of application of the use of the Dirac delta function, as compared with conventional methods of solution. H. Blok, Holland

4372. Sanders, R. W., Transient heat conduction in a melting finite slab: An exact solution, *ARS J.* **30**, 11, 1030–1031 (Tech. Notes), Nov. 1960.

The melting problem for a finite slab with constant thermal properties is considered. It is assumed that the face $x = a$ of the slab is insulated while through the other face the heat input at a rate $H_0(t)$ is flowing in; the molten material is removed immediately, so that the face, initially at the position $x = 0$, moves forward and its position at time t is $S(t)$. This $S(t)$ and the temperature distribution $T(x, t)$ [for $S(t) \leqq x \leqq a$] are to be determined.

The problem is mathematically formulated and transformed into another one with unknown functions $s(t)$ and $\theta(\xi, t)$, with $0 \leqq \xi \leqq 1$ (so that the moving boundary, the essential difficulty of the problem, is eliminated).

Problems like this are to be solved numerically. To get corresponding information for this purpose, namely about dependence of $s(t)$ [resp. $S(t)$] on $M_0(t)$, the indirect problem is solved: ds/dt being chosen, we can find, using separation of variables and hypergeometric functions, exact solution $\theta(\xi, t)$ (in form of an infinite series) and then $M_0(t)$ (depending on the choice of ds/dt). Choosing ds/dt in a simple form with one parameter A and using eventually simple temperature distribution of the slab, author gets a sufficient number of typical problems for comparison. Graphs are presented with results for some selected values of the parameter A. K. Rektorys, Czechoslovakia

4373. Boikov, G. P., and Klyainerman, V. B., Determination of heat energy absorbed by electrical insulation, *Soviet Phys.-Tech. Phys.* **5**, 6, 696–697 (Letters to the Editor), Dec. 1960. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30**, 6, 739–741, June 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4374. Unterberg, W., Simple graphical method for temperature distribution in bodies with linear change in environment temperature, *J. Aerospace Sci.* **28**, 1, 78–79 (Readers' Forum), Jan. 1961.

4375. Noyes, R. N., A fully integrated solution of the problem of laminar or turbulent flow in a tube with arbitrary wall heat flux, *ASME Trans.* **83**C (*J. Heat Transfer*), 1, 96–98 (Tech. Briefs), Feb. 1961.

It is the purpose of this report to derive integrated closed form solution for the variation of the heat-transfer coefficient for the case of fully developed laminar flow in a tube with an arbitrary analytical heat-flux distribution along the length of the tube. From a previous paper and the results reported herein, the case of turbulent flow with arbitrary heat flux may also be treated.

From author's summary

4376. Hartnett, J. P., and Deland, E. C., The influence of Prandtl number on the heat transfer from rotating nonisothermal disks and cones, *ASME Trans.* **83**C (*J. Heat Transfer*), 1, 95–96 (Tech. Briefs), Feb. 1961.

In the present note, additional solutions of the energy equation are reported for the rotating nonisothermal disk or cone with power function surface temperature distribution covering a range of Prandtl number from 0.1 to 100, and values of the exponent m from 0 to 10. From authors' summary

4377. Szentgyorgyi, I., Torok, A., and Szabados, L., A study of heat transfer in suspensions (in Hungarian), *Energia es Atomtechnika* **13**, 9, 388–395, Sept. 1960.

Paper describes an analytical and experimental investigation of the enhancement of heat-transfer coefficients attainable in organic moderators used in nuclear reactors by means of admixing solid particles held in suspension. The organic coolants investigated included various grades of diphenyls, terphenyls and orthophenyls, characterized by poor heat-transfer coefficients and high viscosities. The suspension consisted of finely powdered graphite having a heat-transfer coefficient 2–3 orders of magnitude higher than the organic moderator.

Analytic studies showed that the improvement in heat-transfer coefficient attainable is only 15–25% at a volumetric concentration of 20–25% of the suspended powder. Experimental studies showed an even smaller improvement. Authors conclude that problems incurred by admixing solid particles as suspension in coolant, such as severe erosion, increase in viscosity and larger pumping requirements, negate small improvements attainable in values of heat-transfer coefficient. N. A. Weil, USA

4378. Acrivos, A., A theoretical analysis of laminar natural convection heat transfer to non-Newtonian fluids, *AIChE J.* **6**, 4, 584–590, Dec. 1960.

It is assumed that the non-Newtonian behavior of a liquid can be specified in terms of a "power-law." Thus the coefficient of viscosity is supposed to be proportional to a power of the absolute value of a single component of the rate of shear. In accordance with this rheological behavior the theory of laminar, thermal boundary layers is re-formulated. If Prandtl's number is large and inertia is neglected it is possible to apply dimensional analysis. In this way the heat-transfer coefficient of a number of systems is obtained, in particular for flow past a flat plate and past bodies of cylindrical symmetry. In general it is found that Nusselt's number is proportional to a product of powers of the Grashof and Prandtl numbers. For Newtonian liquids *the exponents are* 1/4 and the factor of proportionality is approximately equal to 0.55. It is not sensitive to the shape of the surface or the exponent in the power law.

Reviewer believes that this otherwise perfect hydrodynamical deduction is limited in its significance by its dependence on the power law. R. Eisenschitz, England

4379. Brown, W. G., The superposition of forced and free convection at low flow rates in a vertical tube (in German), *Forsch. Geb. Ing.-Wes.* (*B*) **26**, 31 pp., 1960.

Theoretical and experimental investigation. Theory is confined to laminar flow. Velocity and temperature distributions are calculated as a function of a parameter Z for the upward and downward flow, and for the two cases $\partial\rho/\partial x \lesseqgtr 0$ (ρ density, x axial coordinate in upward direction). The parameter Z is equal to $(Nu \cdot Gr/4\ Re)^{1/4}$, where the Nusselt, Grashof and Reynolds numbers are referred to tube diameter, mean velocity, temperature and density differences between wall and bulk of the fluid. If $\partial\rho/\partial x < 0$ backflow at the center occurs when $Z > 5$. However, experiments indicate instability beyond $Z = 2$, and decrease of the critical Reynolds number, from 2000 down to 30 for $Z \geqslant 5$. Yet there is satisfactory agreement between measured and computed temperature profiles and between Nusselt numbers, the computed values based upon laminar flow. If $\partial\rho/\partial x > 0$ the flow is much more unstable. Theory is restricted to $Z < 3.6$, while for larger values the flow proved to be turbulent at all Reynolds numbers investigated. For turbulent flow a new semiempirical correlation is proposed, $Nu = 0.0015\ Gr^{2/3}Pr^{1/4}$ where now the fluid temperature at the center is used instead of the bulk temperature.

J. O. Hinze, Holland

4380. Pchelkin, L. M., Heat transfer by natural convection in a vertical pipe (in Russian), Konvektivnyi i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 56–64.

This is an experimental study of heat transfer by natural convection in an electrically heated vertical tube with water and oil as coolants. Nine different tubes, ranging from 0.48 to 1.525 m in length and having outside diameters from 6 to 25 mm, were used in the experiments. Experimental results are tabulated in the paper, and an empirical equation which correlates the data to within ±8% is given by

$$Nu_x = C\,(Pr_f/Pr_w)_x^{1/4}\,(Gr_x \cdot Pr_{fx})^n$$

The constants C and n depend on the flow regime. For laminar flow, $2 \times 10^3 \leqslant Gr_x \cdot Pr_x \leqslant 2 \times 10^{10}$, $C = 0.6$ and $n = 1/4$, but for turbulent flow, $6 \times 10^{10} \leqslant Gr_x \cdot Pr_{fx} \leqslant 3 \times 10^{12}$, $C = 0.150$ and $n = 1/3$. The subscript x refers to the vertical position x. The subscripts f and w refer to the temperatures outside the developing boundary layer and the wall, respectively. Since the Prandtl number range covered in the experiments was from 3.5 to 3×10^3, the factor $(Pr_f/Pr_w)_x^{1/4}$ was introduced to account for the physical property variation with temperature.

The exponent n for both the laminar and turbulent flow regime agrees with the values obtained by earlier investigators. Reviewer feels that the paper would be more valuable if the experimental apparatus and the experimental procedure were discussed.

R. Viskanta, USA

4381. Robbins, W. H., An analysis of thermal radiation heat transfer in a nuclear-rocket nozzle, NASA TN D-586, 24 pp., Jan. 1961.

A computer solution of the radiant heat flux distribution in a nuclear-rocket nozzle is presented which covers a wide range of nozzle geometries, and reactor-face and nozzle-wall temperatures and emissivities. The enclosure is assumed to be made up of gray surfaces which reflect diffusely and the contained gaseous medium is assumed non-absorbing. The reactor-face and nozzle-wall are divided into a number of small finite areas (the exact size and number are not specified); view factors for this system are de-

termined; and the standard radiant heat-transfer equations, using the radiosity concept, are conventionally applied. The analysis does not allow for variations in temperature over the reactor face or nozzle.

In addition to showing the expected effect of temperature, emissivity and nozzle geometry, the results indicate that the heat-flux density is a maximum at the entrance to the nozzle and that nozzle wall temperature has very little effect on the heat flux or distribution.

An approximate method is presented which allows a rapid calculation of the heat-flux distribution in nozzles of different geometries.

T. W. Hoffman, Canada

4382. Binder, J. A., Contribution to the computation of radiant heat transfer surface in arch boilers (in German), *Schiffstechnik* **7**, 39, 211–218, Nov. 1960.

4383. Ungar, E. E., and Mekler, L. A., Tube metal temperatures for structural design, *ASME Trans.* **82** B (*J. Engng. Industry*), 3, 270–276, Aug. 1960.

A procedure is presented for calculating circumferential temperature distributions in tubes exposed to nonuniform radiation. This permits a refinement in the methods of designing tube banks for high-temperature radiant heat exchangers.

R. Gardon, USA

4384. Sparrow, E. M., and Hartnett, J. P., Condensation on a rotating cone, *ASME Trans.* **83** C (*J. Heat Transfer*), 1, 101–102 (Tech. Briefs), Feb. 1961.

This paper presents a theoretical analysis of laminar film condensation on a rotating cone. The results show that the calculated heat-transfer coefficient for the cone does not deviate markedly from the coefficient for the analogous process on a rotating disk. A simple formula is presented relating the condensate layer thickness for the rotating cone and disk.

L. Lapidus, USA

4385. Cole, R., A photographic study of pool boiling in the region of the critical heat flux, *AIChE J.* **6**, 4, 533–538, Dec. 1960.

A photographic study was made to investigate the boiling phenomena in the neighborhood of the critical heat flux. Measurements of bubble diameters, bubble positions relative to the heating surface, local bubble frequencies, and contact angles at known time intervals were obtained from the film. A dimensionless relationship is developed relating bubble velocity, bubble diameter, and contact angle at breakoff. Drag coefficients for freely rising vapor bubbles in saturated liquid are found to be representable by the usual drag coefficient-Reynolds number curves for solid bodies. Jakob's plot of bubble frequency versus bubble diameter at breakoff is extended to high heat-flux values, and a relationship proposed by Deissler at the critical heat flux is found to yield reasonable agreement with the experimental data.

From author's summary by L. S. Dzung, Switzerland

4386. Alad'ev, I. T., and Dodonov, L. D., Critical heat fluxes for boiling of subcooled water in complicated channels (in Russian), Konvektivnyi i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 65–78.

The paper presents results of an experimental investigation of critical (burnout) heat fluxes for water in channels of complicated shape. Three channels, approximating a cluster of fuel pins, were investigated: (a) model of a channel of VVER (Voronezh Atomic Power Station) reactor, (b) annular channel, and (c) channel of VVER with a tight packing of fuel pins. The first two test sections were of such a shape that both the central tube and outer contour of the test section were heated electrically; however, the heat flux applied to the outer contour of the test section was always smaller than at the central tube, and the critical heat flux always occurred at the tube. The design of the test sections is described, but the experimental procedure is not discussed.

The tests were performed at a pressure of 100 atm. The inlet velocity for the first two test sections was approximately one meter per second, and for the third it was from 1.07 to 5.30 m/sec. The subcoolings (based on the exit temperature) ranged from 6.5 to 213°C. No correlations or equations are given in the paper. Results are tabulated; also included are critical heat flux data for tubes obtained in an earlier study; however, the reference to this work is not given. These data cover the pressure range from 20 to 200 atm, inlet velocities from 0.93 to 8.25 m/sec and subcoolings from 0 to 240.6°C.

Reviewer believes that the data given in the paper would be valuable to a pressurized water reactor designer.

R. Viskanta, USA

4387. Dergarabedian, P., Observations on bubble growths in various superheated liquids, *J. Fluid Mech.* **9**, 1, 39–48, Sept. 1960.

The work reported represents an extension of the author's work of some years ago. [*J. Appl. Mech.* **20**, 537–545, Dec. 1953]. Present work includes measurements on the pure organic fluids: ethyl alcohol, methyl alcohol, benzine and carbon tetrachloride, as well as additional measurements on water. The technique has been refined to permit measurement of bubbles down to about 10 microns radius. Bubbles are caused to form and grow in the body of a superheated liquid to which energy is added by means of infrared lamps. Bubble sizes are measured by a photographic technique. The present article is lacking in a description of experimental detail.

Results indicate that, for the liquids investigated, the relative rate of growth of bubbles at a fixed superheat is proportional to $\rho c \sqrt{D}/\rho' L$, where ρ and ρ' are the liquid and vapor densities, c is the specific heat of the liquid, L is the latent heat of vaporization and D, the thermal diffusivity of the superheated liquid. At bubble radii of order 30 microns in water with about 1°C superheat, author observes a relatively long (about 0.1 second) period of nearly constant size followed by a marked increase in growth rate. Author derives the conditions for dynamic stability in terms of equilibrium bubble radii for the case where the vapor pressure remains constant during bubble growth.

T. J. Connolly, USA

4388. Lee, K. S., and Knudsen, J. G., Local shell-side heat transfer coefficients and pressure drop in a tubular heat exchanger with orifice baffles, *AIChE J.* **6**, 4, 669–675, Dec. 1960.

Paper is a portion of a larger project on shell and tube exchangers and correlated data are limited to a 6-in. shell containing four 1-in. tubes with baffles either at 4- or 9-in. spacing. Holes were larger than tube diameters by 1, 2, 3, and 5 sixteenths. Local coefficients to air were measured for several positions between baffles and averages determined by integration. Coefficients varied with distance downstream going through maximum and minimum values. Comparison shows test results about 100% higher than predicted by Short's equation. Pressure drop measurements agree with those of Sullivan and Bergelin. More work will be needed to confirm proposed correlations.

A. C. Mueller, USA

4389. London, A. L., Mitchell, J. W., and Sutherland, W. A., Heat-transfer and flow-friction characteristics of crossed-rod matrices, *ASME Trans.* **82** C (*J. Heat Transfer*), 3, 199–213, Aug. 1960.

Experimental results on heat transfer and pressure loss on flow through crossed-rod matrices over a wide range of Reynolds number are presented in form of graphs. Matrices consisted of 0.375-in. diameter plastic rods arranged in square pattern with the pitch so adjusted as to result in porosity of packing ranging from 0.5 to 0.83. Both the inline and staggered configurations were used. Values of the heat-transfer coefficient were found from the temperature-time history of a single copper rod as it cools within the matrix. The friction coefficient was determined from measurements of pressure loss across the matrix for isothermal flow. No attempt was made to derive empirical equations correlating these coefficients with experimental variables.

This work is a continuation of tests previously done on the random-stacking crossed-rod configurations and reported by Coppage and London [*Chem. Engng. Progress* **52**, 57F–63F, 1956] and Tong and London [*ASME Trans.* **79**, 1558–1570, 1957].

S. Smoleniec, South Africa

4390. Garner, F. H., and Hoffman, J. M., Mass transfer from single solid spheres by free convection, *AIChE J.* **7**, 1, 148–152, Mar. 1961.

Rates of mass transfer by free convection have been measured for spheres of three organic acids dissolving in three solvents, water, *n*-butanol and benzene. The results fell into two groups, one in which flow is completely laminar and one in which turbulence had set in in the boundary layer. Turbulence appears to set in at $N'_{Ra\,crit} \sim 6 \times 10^8$.

From authors' summary

4391. Ralko, A. V., The experimental investigation of heat and mass transfer in the presence of chemical conversions, *Inter. J. Heat Mass Transfer* **1**, 4, 273–279, Jan. 1961.

The results of the experimental investigations on heat and mass transfer in the process of kilning the kaolin and the magnesium hydroxide are adduced in this paper. An experimental installation was developed which allows the temperature field of the heated body, its mass loss and the thermophysical characteristics of the body to be determined. The thermal conductivity coefficients and the coefficients of thermal diffusivity, heat capacity and the heat-transfer coefficient of the mentioned substances were defined in the temperature range from 200° to 1000°C. It was found that the chemical conversions begin at the surface of the capillary porous body and then spread gradually into the interior of the body. The chemical conversion zone deepens into the body according to the parabolic law.

From author's summary

4392. Lebedev, P. D., Heat and mass transfer during the drying of moist materials, *Inter. J. Heat Mass Transfer* **1**, 4, 294–301, Jan. 1961.

Paper deals with heat and mass transfer inside a moist capillary body under conditions of an intensive heating process. It was found by experiments that in the capillaries of a body a pressure of gas and vapor mixture occurs exceeding that of the surrounding air (barometric pressure). One of the main reasons stimulating moisture transfer inside a body in the process of drying is a decrease of this excessive pressure. The coefficients of moisture content are calculated and in analysis the mechanism of drying is given on the basis of the differential equation for heat and mass transfer in dispersed media.

From author's summary by G. Selin, Sweden

4393. Lebedev, P. D., Heat and mass transfer between moist solids and air, *Inter. J. Heat Mass Transfer* **1**, 4, 302–305, Jan. 1961.

Heat transfer during the evaporation of a liquid from porous capillary solids differs from heat transfer during the evaporation of a liquid from a free surface. The Nusselt number depends, in the process of drying of moist solids, not only on the Reynolds number, but also on parametric criteria which characterize the influence of mass transfer on heat transfer. The empirical relations establishing the similarity of criteria for heat and mass transfer are determined on the basis of experimental data.

From author's summary by G. Selin, Sweden

4394. Tailby, S. R., and Portalski, S., The hydrodynamics of liquid films flowing on a vertical surface, *Trans. Inst. Chem. Engrs.* **38**, 6, 324–330, Dec. 1960.

The existence of waves in the wetted wall-column type of apparatus for determining mass-transfer coefficients gives rise to uncertainty in the value of the interfacial area. Measurements for a plane surface of the critical Reynolds number of wave inception and the increase in surface area due to rippling have been found to be in excellent agreement with an extension of a theory put forward by Kapitsa.

G. H. Lean, England

4395. Yee, L., Bailey, H. E., and Woodward, H. T., Ballistic range measurements of stagnation-point heat transfer in air and in carbon dioxide at velocities up to 18,000 feet per second, NASA TN D-777, 36 pp., Mar. 1961.

A new technique for measuring heat-transfer rates in free flight in a ballistic range is described. Measurements of the stagnation-point heat-transfer rates in air and in carbon dioxide at velocities up to 18,000 feet per second are presented. These results are compared with the experimental results obtained in shock tubes and with the results of theory.

From authors' summary

4396. Kunii, D., and Smith, J. M., Heat transfer characteristics of porous rocks: Part 2, Thermal conductivities of unconsolidated particles with flowing fluids, *AIChE J.* **7**, 1, 29–34, Mar. 1961.

Experimental heat transfer studies were carried out in beds of unconsolidated glass beads and sand through which fluids were flowing. The scope of the measurements included four fluids, helium, air, carbon dioxide, and water liquid at atmospheric pressure in beds packed with four sizes of glass beads, 110, 370, 570, and 1,020 μ and with two sizes of sand, 110 and 240 μ. Flow rates ranged from 1 to 26 lb/(hr sq ft) in a direction parallel and countercurrent to energy flow.

The data were interpreted in terms of apparent, effective thermal conductivities of the bed. The values of k_e increase significantly with mass velocity of fluid.

By considering the mechanism of heat transfer in porous media a relationship was developed between k_e and the heat-transfer coefficient between fluid and particle. Treatment of the experimental data in this fashion, combined with available information for larger particles, results in a correlation of Nusselt and Reynolds numbers for air that covers the range $N_{Re_m} = 10^{-1}$ to 10^4.

From authors' summary

4397. Peng, T.-C., and Ahtye, W. F., Experimental and theoretical study of heat conduction for air up to 5000°K, NASA TN D-687, 38 pp., Feb. 1961.

The theoretical value of the integral of thermal conductivity is compared with the experimental values from shock-tube measurements. The particular case considered is the one-dimensional nonsteady flow of heat through air at constant pressure. This approach has been previously described in NASA TR R-27. However, the correlation between theory and experiment was uncertain because of the large scatter in the experimental data. In this paper, an attempt is made to improve the correlation by use of a more refined calculation of the integral of thermal conductivity, and by use of improved experimental techniques and instrumenta-

tion. As a result of these changes, a much closer correlation is shown between the experimental and theoretical heat-flux potentials. This indicates that the predicted values of the coefficient of thermal conductivity for high-temperature air may be suitably accurate for many engineering needs, up to the limits of the test (4699 °K). From authors' summary

4398. Maulard, J., Problems arising in measurement of heat flux for kinetic heating phenomena (in French), Rech. Aéro. no. 77, 29–41, July/Aug. 1960.

Paper analyzes various methods for the measurement of heat flow. It concludes that it is virtually impossible to achieve absolute calibration and thus assess the reliability of different measuring appliances. Comparative measurements of a common heat flow are the only possible check and these suggest that under the best conditions accuracies of the order of 5% can be achieved.

W. S. Hemp, USA

4399. Vertman, A. A., and Samarin, A. M., Viscosity of liquid nickel and its alloys with copper, *Soviet Phys.-Doklady* **5**, 3, 598–601, Nov./Dec. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **132**, 3, 572–575, May 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4400. Batisheher, Ya. F., An investigation of the heat loss from the wall to the atmosphere, of a pipe filled with steel balls. Comparison of the heat emission from a pipe with and without a filling. The general case of heat transfer from ducts (in Russian), *Trudi Novocherk. Politekhn. In-ta* no. 70/88, 33–43, 65–69, 71–78, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 3951.

An experimental determination has been made of the mean coefficient of heat emission from the heated air in a circular pipe filled with round steel balls. The pipe diameter, $D = 31.5$ mm, experimental length $L = 518$ mm; ratio of pipe diameter to ball diameter variable between $D/d = 2.2$ to 12.84; Reynolds number R between 7.62 and 10,600. A critical evaluation of the experimental results has enabled a relationship of the form

$$N_D = c\left(\frac{D}{d}\right)^m \overline{R}_d{}^n$$

to be established for three regions corresponding to laminar, transitional, and turbulent flow. The boundaries of these regions in regard to R_d coincide with the boundaries for smooth ducts. In a turbulent flow, the heat emission from pipes with a filling is approximately ten times that from a smooth pipe for the same values of R_D. By introducing corresponding determining dimensional terms and suitable evaluation of the experimental values, generalized expressions have been derived for the three flow regions, applicable to both filled and smooth pipes.

L. G. Genin

Courtesy Referativnyi Zhurnal, USSR

Combustion

(See also Revs. 4231, 4414)

4401. Kinbara, T., and Akita, K., An approximate solution of the equation for self-ignition, *Combustion and Flame* **4**, 2, 173–180, June 1960.

The partial differential equation

$$\frac{\partial \theta}{\partial \tau} = \nabla^2 \theta + \delta e^{\theta}$$

which arises in the theory of self-ignition cannot be solved in closed form. The authors obtain an approximate solution by replacing e^{θ} by $1 + \beta\theta$. The value of β is chosen by comparing the solutions of

$$\nabla^2 \theta + \delta e^{\theta} = 0$$

and

$$\nabla^2 \theta + \delta (1 + \beta\theta) = 0$$

The approximate solution agrees qualitatively with experimental measurements of the temperature history of piles of sawdust. The quantitative discrepancies are attributed to the neglect of fuel consumption in the formulation of the basic equation.

W. Squire, USA

4402. Detonation and its initiation, Seventh Symposium (International) on Combustion, London and Oxford, Aug. 28–Sept. 3, 1958; New York, Academic Press, 1959, 799–880.

Part IX, on Detonation and its Initiation, contains eight papers, seven of which concern the transition from flame to detonation in gases, the other being on the shock initiation of condensed phase explosives. A paper by Brinkley and Lewis reviews briefly the existing state of experiment and theory. Three papers deal with the theoretical analysis of the gas motion set up by the flame and the formation of shock waves. Two of these make detailed and quantitative analyses of the experimental flame wave-speed photographs which suggest that the gas dynamic problem is well understood even though the processes which lead to flame acceleration are less well so. The three experimental papers present some remarkable pictures of flame shapes in the pre-detonation period which support the view that the major factors causing accelerated burning are the turbulent boundary layer at the gas-wall interface ahead of the flame and the influence of the velocity gradient near the wall on flame shape. In one of these papers D. R. White shows some interferograms of steady detonation waves in hydrogen oxygen mixtures initiated by shock waves which reveal a local nonplanar and oscillatory motion at the front in strong contrast to the sharp plane front of the nonreactive shock.

The remaining paper is almost entirely devoted to the shock initiation of condensed phase explosives studied by framing camera techniques under marginal initiation conditions. Three of the papers, which are all in English, are by Soviet authors.

G. K. Adams, England

4403. Eckhaus, W., Theory of flame-front stability, *J. Fluid Mech.* **10**, 1, 80–100, Feb. 1961.

Paper is summary of author's work at M.I.T. attempting to find theoretical explanation of cellular flames as observed by Markstein. Earlier theories of Landau and Markstein are superseded by consideration of perturbations of flame velocity, diffusion and heat conduction but work is restricted by present inadequate understanding of even steady-state one-dimensional flame propagation. Assuming flame thickness small compared with cell wavelength, stability is shown theoretically dependent on mixture properties. Results are in broad qualitative agreement with Markstein's experimental observations that cellular instability generally occurs only with rich mixtures. D. G. Stewart, Australia

4404. Povinelli, L. A., A review of turbulent flame propagation (in Italian), *Aerotecnica* **40**, 5, 272–286, Oct. 1960.

Combustion occurring in practical propulsion devices invariably takes place under turbulent conditions. Unfortunately, it is difficult to make quantitative predictions concerning the effect of the turbulence properties on the increase of the normal or laminar burning velocity. This paper presents a review of the progress in turbulent flame theories for homogeneous gases. In first part, the flame theories and corresponding flame models are discussed as well as their experimental verification. The second part of the

paper is concerned with the most recent developments in turbulent flame theories and a criterion for the flame model, for which some interesting combustion data are presented.

From author's summary

4405. Tsuji, H., Ignition and flame stabilization of stream of combustible gaseous mixtures by hot jet, Aero. Res. Inst., Tokyo Univ. Rep. 357, 119–136, June 1960.

Development of combustion in a two-dimensional mixing zone between a semi-infinite stream of a gaseous combustible mixture and a hot jet of combustion products of a finite width is studied by the solution of boundary-layer equations. Approximate analytic expressions are developed for the temperature and the concentration profiles.

Numerical results are presented for the thermal decomposition of azomethane and hydrazine. Author finds that there is a minimum width of the hot jet necessary for ignition of the combustible mixture. This minimum width decreases as the temperatures of the combustible gas and the hot jet increase.

T. Y. Toong, USA

4406. Filippi, F., Fabbrovich-Mazza, Laura, and Calandrino, S., Studies of bluff-body flame stabilization: Part 2, Effect of auxiliary gas injection into recirculation zone (in Italian), *Aerotecnica* **40**, 1, 25–33, Feb. 1960.

Experimental research has been performed from 1957–1959 with the aim of improving and eventually controlling the stability limits of bluff-body flameholders. In order to do this small amounts of auxiliary gases (methane, propane, air, nitrogen) were injected through a single 5-mm rod flameholder in a two-dimensional burner supplied with premixed methane/air and propane/air mixtures.

The results show the same general trend as reported by other independent researchers. The injection of fuel shifts the stability limits toward leaner mixtures; the injection of air shifts these limits toward richer mixtures.

With fuel injection the maximum blow-off velocity decreases with increasing injection rates. This unexpected, and previously unreported, phenomenon suggests some reconsideration of our views on the mechanism of flame stabilization and blowoff. An extension of the current Zukoski hypothesis is proposed.

From authors' summary

4407. Cheng, S. I., and Chiu, H. H., Mixing and chemical reaction in an initially non-uniform temperature field, *Inter. J. Heat Mass Transfer* **1**, 4, 280–293, Jan. 1961.

This paper presents a continuation of the senior author's work on combustion in a constant-pressure mixing region behind a flat plate. As in the previous work, the boundary-layer approximation is applied to a flow in which both streams initially have a Blasius velocity profile and one stream is composed of hot products of combustion while the other stream contains cool premixed reactants. The novel aspect of the present problem is that the temperature profile at the trailing edge is permitted to vary in an arbitrary smooth manner; previous treatments assumed a (different) constant temperature in each approach stream. The solution for the temperature field is obtained as an expansion in powers of a parameter measuring the distance downstream from the trailing edge; terms through third order are retained in the expansion. The results (valid in the vicinity of the trailing edge) are given in terms of "universal functions" (obtained from numerical solutions of linear differential equations) which facilitate the inclusion of arbitrary initial temperature distributions. An interesting prediction of the theory is that, even in the absence of chemical reactions, the temperature gradient normal to the dividing streamline *increases* for a short distance downstream because the inward convection of hot and cold material overbalances the effect of thermal conduction. This phenomenon was verified by experiments with hot and cold air streams. In a typical case involving combustion, the theory predicts that the local bulge in the temperature profile first develops at a position slightly farther upstream and much farther into the hot stream than it does for the case in which the initial temperature profile is a step function.

Undefined notation and an extraordinarily large number of misprints make the paper difficult to read.

F. A. Williams, USA

4408. Kling, R., Experimental investigation of the phenomena of combustion in a sector of an annular combustion chamber of a turbojet engine (in German), *Z. Flugwiss.* **8**, 12, 345–352, Dec. 1960.

Author describes tests conducted on a sector of an annular combustion chamber such as used for turbojet engines. The sector tested included four combustion nozzles and represents 1/5 of the total combustion system of a typical engine. By means of high-speed microphotography, the distribution and behavior of the fuel droplets were studied. The flame was analyzed by spectroscopy.

The effects of several parameters on combustion efficiency were investigated. Tests were limited to simulated conditions at an altitude of 50,000 feet.

H. Hegetschweiler, USA

4409. Gutman, V. R., Solid propellent burning rate theory; recently advanced theories, their limitations and possible areas for future investigation, *Aircr. Engng.* **32**, 379, 255–260, Sept. 1960.

The progress of solid propellent technology appears to have been retarded by lack of development of a fundamental mechanism of burning. A study of previous work indicates that while experimental techniques used are valid, hypotheses were inadequate; fresh hypothetical approaches are needed. There is evidence of lack of theory development in the more fundamental field of the combustion of turbulent, pre-mixed, fuel-rich flames as it applies to propellent burning. The roles of radiative heat transfer and a physical disintegrative mode of surface dissipation are proposed for consideration. Previous experimental techniques together with new ones are proposed to exploit these hypotheses.

From author's summary

4410. Magnusson, U., and Brynander, B., A general method for the computation of equilibrium compositions of chemical reactions at high temperatures on a high-speed digital computer, aimed at the construction of enthalpy-entropy diagrams (in Swedish), *Astronautik, Sweden* **2**, 2, 61–88, 1960.

A set of equations constituting a general method for the computation of chemical equilibrium in gaseous or partly condensed systems is deduced in detail and a program for the high-speed solution of these equations on a Pegasus digital computer is described. The program handles simultaneously up to 36 gaseous and liquid or solid reaction products of, at the same time, not more than 8 different chemical elements. It is specifically aimed at the subsequent construction of enthalpy-entropy diagrams up to 6000 °K. At each calculated point the program provides: equilibrium compositions, temperature, entropy, enthalpy, pressure, average molecular weight, and average specific heat at constant pressure.

From authors' summary

4411. Robotti, A. C., The choice of propellants for aircraft rocket engines (in Italian), *Aerotecnica* **40**, 1, 47–55, Feb. 1960.

The general considerations which guide the choice of propellants for aircraft rocket engines are summarized and the trend of the American, British, and French designers toward different propellant combinations, in particular, different oxidizers, is discussed.

From author's summary

4412. Fox, M. D., and Weinberg, F. J., Optical methods for the study of flames in turbulent pre-mixed gas streams, *Brit. J. Appl. Phys.* **11**, 7, 269–273, July 1960.

Three optical methods for the investigation of flame processes in turbulent gases are described. All are based on ray deflection by the steep refractive-index gradient occasioned by large temperature and composition changes across the flame front. This is utilized in three different optical systems designed (1) to give a measure of the randomness of orientation of the fluctuating flame front in any locality; (2) to delineate the instantaneous flame-front surface; and (3) to map time-mean deflection (and hence optical path) distributions. Their purpose and use is discussed and illustrated by examples of the records obtained.

From authors' summary

Prime Movers and Propulsion Devices

(See also Revs. 4270, 4381, 4388, 4409)

Book—4413. Shepherd, D. G., Introduction to the gas turbine, 2nd ed., New York, D. Van Nostrand Co., Inc., 1960, ix + 300 pp. $7.75.

Author has completely re-written the first edition of "Introduction to the gas turbine." The reader is assumed to have an elementary knowledge of thermodynamics and fluid mechanics.

Analysis of gas-turbine cycles, including graphical method of Hawthorne and Davies, is well done. General chapter on compressors and turbines includes brief treatment of several topics, energy transfer, efficiency, stagnation conditions, one-dimensional compressible flow, radial equilibrium, etc. Of subsequent chapters, those on combustion and heat transfer are particularly useful surveys of fundamentals. Those on turbines, axial and centrifugal compressors provide the reader with useful summaries of present-day practice.

Reviewer would have liked to see more critical analysis of turbine utilization factor, fuller analysis of three-dimensional flows, but in a book surveying the whole field of the gas turbine it is difficult to delve more deeply than Shepherd has done.

The book is largely descriptive, rather than analytical, and like the first edition is clearly written. It will be more useful as a teaching book than as a volume for the research worker; more useful for the beginner than the expert in the field. Author has given an improved version of a most useful book.

J. H. Horlock, England

Book—4414. Feodosiev, V. I., and Siniarev, G. B., Introduction to rocket technology (Translated from the Russian by S. N. Samburoff), New York, Academic Press, 1959, x + 344 pp. $9.50.

Textbook is intended for advanced college level to give comprehensive summary of engineering fields contributing to rocket technology. With so much material to be covered, authors do not go into much detail, and book is most useful as an introduction. Scope can be indicated by chapter headings.

1. Basic relationships in the theory of reactive motion.
2. Types of jet propelled aircraft and their basic construction.
3. Types of reaction motors, their construction and operational characteristics.
4. Rocket motor fuels (liquid, solid, and nuclear).
5. The processes in the combustion chamber of a rocket motor.
6. Flow of the combustion products through the nozzle of a rocket motor.
7. Forces and moments acting on the rocket in flight.
8. Rocket flight trajectory (includes flight beyond Earth's atmosphere but still in gravitational field).
9. Basic principles of stabilization and steering (includes discussion of gyroscopic devices).
10. Ground equipment and launching devices

This book is complementary to Sutton's "Rocket propulsion elements,"[Wiley,1956; AMR **10** (1957), Rev. 581] which covers actual propulsion aspects in greater detail. It is interesting to compare the historical sections of each book and the emphasis on who did what.

W. F. Davis, USA

4415. Henneberry, H. M., Corrington, L. C., and Becker, R. D., A thermodynamic analysis of thrust augmentation for nuclear rockets, NASA TN D-581, 30 pp., Mar. 1961.

Thrust augmentation of the nuclear rocket system was obtained by the addition of chemical power. A liquid oxidant, oxygen or fluorine, was added to burn with the heated hydrogen propellant issuing from the reactor. The specific impulse of the nuclear rocket afterburner was optimized by the addition of liquid hydrogen along with the oxidant. Data are presented for hydrogen reactor exit temperatures from 3500 ° to 6500 ° R. Comparison with other chemical thrust augmentation systems described herein indicates that the optimum afterburner is the most efficient method of chemical thrust augmentation for nuclear rockets.

From authors' summary

4416. Dunning, J. E. P., The optimum size of a rocket engine, *J. Roy. Aero. Soc.* **64**, 600, 717–742, Dec. 1960.

Specific weight of liquid bi-propellent rocket engine is calculated as function of size on the basis of special premises and assumptions concerning effects of sealing on component efficiencies. Detailed consideration is given to departures from geometrical scaling which are dictated by combustion requirements or by other practical considerations. Reductions in specific impulse as scale is reduced, resulting from decreased pump efficiencies, reduced transit time through propelling nozzle, increased frictional losses, etc., are debited as effective increases in engine weight, assuming that 1% loss in specific impulse is equivalent to 10% increase in engine weight. Resulting deviation from the "square-cube law" which would relate thrust to weight in case of perfect dimensional similarity leads, within selected premises, to a minimum specific weight at thrust level near 50,000lb.

J. V. Foa, USA

4417. Ruston, G., The reliability of multiple rocket engines, *Aero. Quart.* **11**, 4, 371–386, Nov. 1960.

Author constructs a mathematical model relating reliability, development time and number of engines. From this model one may presumably make the choice between multiple-engine boosters and single-engine boosters for a given application and development schedule. Author concludes that multiple engines might be preferable for a short development schedule.

Since the elements of the model used have only qualitative validity, any quantitative decision based on the theory seems unwarranted at present. As the author notes, the validity of the model would be greatly improved by evaluation of its parameters from actual development experience. The functional relations chosen to relate reliability, thrust, development time, etc., do not reflect some important practical situations. An example is the discontinuity in development time versus number of engines as thrust of individual engines of a set is reduced to that of an existing, fully developed engine. The absence of explicit consideration of the relative weights or cost of single- and multiple-engine systems will prove unnerving to development engineers.

E. W. Price, USA

4418. Giorgieri, L., Ramjet versus rocket in long range vehicles (in Italian), *Aerotecnica* **40**, 1, 34–38, Feb. 1960.

A method is presented for comparing the convenience of using either ballistic or aerodynamic lift in long range vehicles.

It is noted that the main parameter for comparison is the aerodynamic efficiency and author analyzes for which value of aerodynamic efficiency, the aerodynamic lift allows longer ranges than ballistic rockets, when over-all weight is the same.

Advantages in using ramjets (and specially steady-state-detonative-combustion-ramjets) against ballistic rockets are considered.

From author's summary

4419. Pascucci, L., On the possibilities of the ramjet in very high velocity propulsion (in Italian), *Aerotecnica* **40**, 1, 39–46, Feb. 1960.

The possibilities and structural features of the hypersonic ramjet are discussed. The performance of said engine is calculated using some special thermodynamical diagrams for combustion gases in chemical equilibrium. Some considerations on the utilization of the fuel chemical energy and the area regulation are made. From author's summary

4420. Sacerdote, U., Definition of the forces acting on an aerodynamic propulsive ducted body (in Italian), *Aerotecnica* **40**, 1, 56–63, Feb. 1960.

From the analysis of the elementary forces acting on an aerodynamic propulsive ducted body expressions of the propulsive thrust and of the external drag are obtained. By comparison of said expressions with the conventional definition of the thrust of an air-breathing-jet-engine, the definitions of "momentum drag." "additive drag" and "scoop incremental drag" are derived. The formulas thus obtained are applied to the problem of the evaluation of the external drag of a ducted body in a wind tunnel; the analysis is performed also in the case in which the model is sting supported and the aerodynamic forces are measured through an internal balance. From author's summary

4421. Johnson, P. G., and Smith, R. L., An optimization of powerplant parameters for orbital-launch nuclear rockets, NASA TN D-675, 47 pp., Feb. 1961.

Performance of orbital-launch nuclear rockets has been computed for ranges of gross weight, mission energy, and hydrogen temperature to show the effects of variations in hydrogen pressure, reactor power, and reactor flow area. Spacecraft performance is shown to be relatively insensitive to variations in design-point pressure or power. Pumped systems give maximum residual load at reactor-exit pressures of 100 to 300 lb/sq in. abs and powers of 300 to 1500 megawatts for gross weights from 30,000 to 150,000 pounds. Reductions in pressure or power by factors of 2.5 to 5.0 correspond to only a 5% reduction in residual load. Pressurized systems optimize at 30 to 35 lb/sq in. abs and 80 to 400 megawatts for the same gross-weight range, with 95% performance corresponding to pressure or power reduction by factors of 1.5 to 2.5. From authors' summary

4422. Swanson, B. W., Somers, E. V., and Heikes, R. R., Optimization of a sandwiched thermoelectric device, *ASME Trans.* **83** C (*J. Heat Transfer*), 1, 77–82, Feb. 1961.

In this paper an optimum geometry for a sandwiched thermoelectric device is calculated. An optimum is said to be found when the maximum thermal efficiency is computed, subject to the constraints of interface temperatures, over-all length, area and the given material properties. The length of the individual sandwich elements is calculated as a consequence of the optimization.

The calculation follows classical procedures and is carried out by an iteration scheme using an automatic computer. In evaluating the first derivative of the efficiency it is found necessary to neglect some "small" terms. Since the maximum is assumed to exist where its first derivative vanishes the reader would expect that the values of the independent variable at the maximum would be in error. The calculations given in the paper under discussion support this expectation. Fortunately the maximum is very broad in the cases calculated.

The reviewer believes the formal procedure to be adequate, however for a particular design additional physical data are required to complete the optimization.

It may be of interest to note that a program of the kind given here could be used to estimate the effects of conductors on thermocouple installation. E. E. Covert, USA

Magneto-fluid-dynamics

Book—4423. Harris, L. P., Hydromagnetic channel flows, New York, John Wiley & Sons, Inc., 1960, vi + 90 pp. $2.75.

Author considers flows of viscous, incompressible electrically-conducting fluids in rectangular channels subjected to transverse magnetic fields. Following topics are treated: Study of hydromagnetic turbulence; turbulence in hydrodynamic channel flows; Mathematical properties of a turbulent D-C hydromagnetic flow; Dimensional analysis for a turbulent hydromagnetic flow; Experimental results for a turbulent D-C hydromagnetic flow; Correlation of theory and experiment; Further deductions from the theory; Approximate analysis of a laminar induction-driven flow; Turbulent induction-driven flow; Conclusions and suggestions for further work. Techniques developed are analogous to the "semiempirical" methods of modern fluid mechanics, with combined use of basic mathematical laws, dimensional analysis and experiments.

R. Nardini, Italy

4424. Coburn, N., and Ong, R. S. B., The characteristic system for three-dimensional, steady, isentropic, compressible, hydromagnetic flows, *J. Math. Mech.* **9**, 5, 715–732, Sept. 1960.

Paper derives the characteristic system for steady three-dimensional isentropic compressible hydromagnetic flow and the properties of its normals. The normal cone exists for all such flows and depends on Mach and Hartmann numbers; it is analyzed in arbitrary curvilinear coordinates. L. Trilling, USA

4425. Uflyand, Ya. S., On some cases of nonequilibrium flow of a conducting liquid in an annular tube, *Soviet Phys.-Tech. Phys.* **5**, 7, 753–757, Jan. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30**, 7, 799–802, July 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4426. Dzhorbenadze, N. P., and Sharikadze, D. V., The flow of a conducting viscous fluid between two porous planes, *Soviet Phys.-Doklady* **5**, 4, 673–676, Jan./Feb. 1961. (Translation of *Dokladi Akad. Nauk SSSR (N. S.)* **133**, 2, 299–302, July 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

In the present work authors consider the flow of a viscous incompressible, electrically conducting fluid in the space between two parallel porous planes, when there is a homogeneous constant external magnetic field H_0 acting at right angles to the planes.

From authors' summary

4427. Greenspan, H. P., On the flow of a viscous electrically conducting fluid, *Quart. Appl. Math.* **18**, 4, 408–411 (Notes), Jan. 1961.

Two-dimensional perturbations of a parallel flow with coincident constant magnetic intensity are considered. Linearized equations admit similarity of flow. Resulting total differential equations can be solved to give flows around parabolic cylinders with or without injection of conducting fluid. Flow field of a flat plate with injection is given as an example. M. S. Uberoi, USA

4428. Chekmarev, L. G., Some problems of stationary flow of a conducting liquid in an infinitely long annular tube in the presence of radial magnetic field, *Soviet Phys.-Tech. Phys.* **5,** 6, 565–569, Dec. 1960. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 6, 601–605, June 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

In this paper authors consider three special types of stationary flow of an incompressible viscous conducting liquid in an infinitely long annular tube in the presence of radial magnetic field. Exact solutions of the equations of magnetohydrodynamics are obtained for the cases considered. From author's summary

4429. Uflyand, Ya. S., and Chekmarev, I. B., Investigation of transient flow of a conducting fluid in a plane channel with moving walls, *Soviet Phys.-Tech. Phys.* **5,** 5, 437–442, Nov. 1960. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 5, 465–471, May 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

In the present paper authors present an exact solution for the one-dimensional transient magnetohydrodynamic problem of a plane-parallel layer in a transverse magnetic field where the walls which bound the layer move with given velocities. It is shown that in the solution of problems of this kind it is necessary to consider the induced currents which flow in the medium which surround the fluid (channel walls). It is also shown that problems of this kind are related to certain boundary-value problems which are characterized by a mixed eigenvalue spectrum.

From authors' summary

4430. Gupta, A. S., Flow of an electrically conducting fluid past a porous flat plate in the presence of a transverse magnetic field (in English), *ZAMP* **11,** 1, 43–50, 1960.

The effect of a transverse field on the flow past an infinite plate subject to suction or injection is investigated. A magnetic pressure number S is defined, and simple solutions of the steady hydromagnetic equations are obtained in the case $S > 1$. An indeterminancy appears when $S < 1$. The heat-transfer equation is integrated. Reviewer feels that a physical interpretation of these solutions in terms of vorticity diffusion would have been illuminating. H. K. Moffatt, England

4431. Williams, W. E., Some solutions for steady linearised magnetohydrodynamic flows, *Appl. Scient. Res.* (*A*) **9,** 6, 424–428, 1960.

Paper considers the theory of steady flow past small obstacles of an incompressible electrically conducting fluid in the presence of a magnetic field. The solutions of the linearized flow equations are found in terms of two independent scalar functions and can be applied to fluids of both finite and infinite conductivity. The magnetic field can be either parallel or perpendicular to the flow direction. An example is given for flow past a wavy wall, and in the limit of infinite conductivity the results are in agreement with those of Resler and Sears [*J. Fluid Mech.* **5,** 1959; AMR **12** (1959), Rev. 5255]. R. Siegel, USA

4432. Kuzantsev, A. P., Flow of a conducting gas past a current-carrying plate, *Soviet Phys.-Doklady* **5,** 4, 771–773, Jan./Feb. 1961. (Translation of *Doklady Akad. Nauk SSSR (N. S.)* **133,** 2, 318–320, July 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4433. Kakutani, T., Effect of transverse magnetic field on the flow due to an oscillating flat plate: Part 2, *J. Phys. Soc. Japan* **15,** 7, 1316–1331, July 1960.

Paper analyzes Rayleigh's problem for a perfectly conducting oscillating flat plate in an incompressible electrically conducting viscous fluid in the presence of a magnetic field parallel to the plate. Solutions in terms of magnetic and hydrodynamic Reynolds numbers and Alfvèn number are presented. Drag increases and oscillation phase lags distinguish the solution from the classical solution. L. Trilling, USA

4434. Pavlov, K. B., Some properties of stationary flows in magnetic gas dynamics, *Soviet Phys.-JETP* **12,** 2, 216–218, Feb. 1961. (Translation of *Zh. Eksp. Teor. Fiz.*, USSR **39,** 2 (8), 304–307, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

Stationary quasi-one-dimensional and one-dimensional flows are considered. It is shown that shock wave formation is possible under certain conditions in the case of media with finite conductivity. From author's summary

4435. Vandakurov, Yu. V., Stationary state of a think circular plasma pinch of finite conductivity, *Soviet Phys.-Tech. Phys.* **5,** 9, 1069–1071, Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 9, 1134–1136, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4436. Regirer, S. A., On an exact solution of the equations of magnetohydrodynamics, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **24,** 2, 556–561, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Hartman (Hg-Dynamics, I. *Kgl. Danske Vidensk. Selskab. Math.-Fys. Medd.* **15,** 6, 1937) investigated the flow of a viscous electrically-conducting fluid between parallel plane walls under the conditions that all the parameters are unchanged in the direction of the stream. The author solves the magnetohydrodynamic system without those restrictions. Starting from the usual system of four equations in three unknowns (p, V, H) with constant density, author introduces the vector potential A of the magnetic field, and obtains two equations which have to be simultaneously satisfied by A. This allows the author to calculate the magnitude of the velocity. Next, the author calculates the intensity of the magnetic field and the pressure. As a particular case the author calculates the flow in a half-space. It is assumed that the values of the velocity and of the longitudinal component of the magnetic field are given on the boundary of the half-space and at infinity. With this the author calculates the velocity and magnetic field intensity satisfying boundary conditions. In conclusion the author emphasizes that the main goal of the present paper is to find plane magnetic fields whose presence makes it possible for the plane rectilinear motion of a fluid to exist. The author found one such field, which obviously belongs to a wide class of exact solutions of the equations of magnetohydrodynamics. M. Z. v. Krzywoblocki, USA

4437. Lur'e, K. A., On a class of plane problems in magnetohydrodynamics, *Soviet Phys.-Tech. Phys.* **5,** 6, 691–692 (Brief Communications), Dec. 1960. (Translation fo *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 6, 736–738, June 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4438. Talwar, S. P., Stability of a conducting rotating fluid of variable density, *J. Fluid Mech.* **9,** 4, 581–592, Dec. 1960.

Author investigates the Rayleigh instability of an incompressible, infinitely electrically conducting, inviscid fluid of variable density under the influence of an horizontal magnetic field and Coriolis forces. The configuration rotates uniformly with an angular velocity about the Z-axis. The considered equations are those of momentum, continuity and Maxwell. To investigate the stability of the static equilibrium configuration characterized by the condition velocity = 0, the author follows the standard procedure; he considers the effect of a small velocity field disturbance. The equations, so linearized, are combined, giving one final equation for the velocity component w. All the components of the disturb-

ance vary with t as exp (nt); the density and the permanent horizontal magnetic field are assumed to be stratified in the upward direction.

Author first considers some special subcases. (a) Nonrotating configuration: author provides formulas for n, phase velocity of the propagation of the horizontal waves, the group velocity, and the conditions for the stability. (b) rotating configuration without magnetic field: the formula for n, conditions for stability. (c) Simultaneous presence of rotation and magnetic field: there are provided results analogous to those given above. As the next case the author investigates the hydromagnetic stability of two superposed rotating fluids with a few subcases: Nonrotating superposed fluids with magnetic field; rotating superposed fluids with zero magnetic field; homogeneous fluid under the joint influence of rotation and field; instability in superposed fluids under joint influence of rotation and field with two rotations: low and high.

M. Z. v. Krzywoblocki, USA

4439. Roberts, P. H., and Tatsumi, T., The decay of magnetohydrodynamic turbulence, *J. Math. Mech.* **9,** 5, 697–713, Sept. 1960.

Making use of the approximation of zero fourth-order cumulants, authors discuss the variation in the kinetic and magnetic energy spectrum functions in decaying isotropic magnetohydrodynamic turbulence. In particular, they consider what happens when energy is initially contained within a particular range of eddy sizes, and derive the limiting ratio of magnetic to kinetic energies for small Reynolds numbers.

N. Curle, England

4440. Mikhailov, V. I., The scattering of electromagnetic waves over turbulent pulsations with consideration for a vortex of mean velocity (in Russian), *Ukr. Fiz. Zb.* **3,** 3, 351–357, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10313.

It is proposed to bring in a term, describing the transfer of energy from the vortex-affected mean motion to the turbulent, into the equation for the carrying-over of energy by means of the turbulence spectrum [*see* Batchelor, "Theory of homogeneous turbulence," Izd-vo in Lit., 1955]. This term is derived either (a) in accordance with Geisenberg's spectral theory of turbulence, or (b) in agreement with Obukhov's spectral theory. The paper continues by describing the finding of a stepped formula for the spectral density of energy in both cases; here the above-mentioned term has a basic value, on the assumption that a region of spectral turbulence exists. The obtained spectra for the pulsation velocity are utilized for finding the spectra for the pressure pulsations, while these, in their turn—analogously to the procedure adopted in the work of Villars and Weisskopf [F. Villars, V. F. Weisskopf, *Phys. Rev.* **94,** 2, 232–240, 1954]—are used for the calculations of the effective diameters for the scattering of the radio waves.

V. I. Tatarskii

Courtesy Referativnyi Zhurnal, USSR

4441. Dokuchaev, V. P., On the growth of magnetohydrodynamic waves in a plasma stream moving through an ionized gas, *Soviet Phys.-JETP* **12,** 2, 292–293, Feb. 1961. (Translation of *Zh. Eksp. Teor. Fiz., SSSR* **39,** 2 (8), 413–415, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

A dispersion equation describes the propagation of electromagnetic waves in a plasma stream moving through an ionized gas. The low-frequency case (below the ion gyrofrequency), when the waves degenerate into magnetohydrodynamic waves, is considered in detail. It is shown that the system becomes unstable if the flow velocity exceeds the velocity of Alfvén waves in the stream-plus-stationary-plasma system. In this case one of the normal waves builds up with time.

From author's summary

4442. Polovin, R. V., Contribution to the theory of simple magnetohydrodynamic waves, *Soviet Phys.-JETP* **12,** 2, 326–330, Feb. 1961. (Translation of *Zh. Eksp. Teor. Fiz., SSSR* **39,** 2 (8), 463–470, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

The Riemann invariants are computed for simple magnetohydrodynamic waves. The change of velocity in fast and slow magnetoacoustic waves is determined for the case when the magnetic pressure in front of the wave is much smaller than the hydrostatic pressure.

From author's summary

4443. Berezin, O. A., Self-simulating motion of a gas with plane waves in magnetohydrodynamics, *Soviet Phys.-Doklady* **5,** 4, 670–672, Jan./Feb. 1961. (Translation of *Doklady Akad. Nauk SSSR (N. S.)* **133,** 2, 296–298, July 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4444. Korobeinikov, V. P., and Karlikov, V. P., On the interaction of strong explosion waves with an electromagnetic field, *Soviet Phys.-Doklady* **5,** 4, 679–683, Jan./Feb. 1961. (Translation of *Doklady Akad. Nauk SSSR (N. S.)* **133,** 4, 764–767, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

In this note the problem of a strong point explosion in a medium with infinite conductivity in the presence of a weak magnetic field is investigated, as is also the question of the interaction of plane shock waves, ionizing the gas, with a weak electromagnetic field.

From authors' summary

4445. Tkalich, V. S., Finite-amplitude waves in a multicomponent conduction medium, *Soviet Phys.-JETP* **12,** 1, 52–55, Jan. 1961. (Translation of *Zh. Eksp. Teor. Fiz., SSSR* **39,** 1 (7), 73–77, July 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

The set of equations for a nonideal plasma in the hydrodynamic approximation is reduced to a linear one without assuming that the signal is small. Propagation of finite amplitude waves in the presence of a magnetic field is investigated; a comparison is made with well-known results obtained on the basis of other assumptions.

From author's summary

4446. Lighthill, M. J., Studies on magneto-hydrodynamic waves and other anistropic wave motion, *Phil. Trans. Roy. Soc. Lond.* (*A*) **252,** 1014, 397–430, Mar. 1960.

The equations of magnetohydrodynamics of a compressible, perfectly conducting fluid, neglecting viscosity and heat conduction, have been linearized by perturbing a constant magnetic field using the acoustic approximation, and the equations for the propagation of X-component of vorticity, dilatation and X-component of strain-rate are derived. These are partial differential equations with constant coefficients. The waves due to a point source of finite and infinitesimal extent are considered and attention is primarily focused on sources of fixed frequency. The asymptotic solutions are obtained satisfying the radiation condition, which represent waves going out from the source. The mathematical results are given a partial physical interpretation by determining the velocity of energy propagation in a plane wave traversing an anisotropic medium. These show that even for a nondispersive medium, the energy propagation velocity is not in general normal to the wave fronts, although its component normal to them is the phase velocity.

It is also shown that the magnetohydrodynamic waves in an incompressible, inviscid and perfectly conducting fluid propagate in one direction only, unattenuated along the magnetic lines. However, when compressibility and Hall current are present the propagation is no longer unidirectional and attenuation-less.

Reviewer thinks that the paper is an important contribution to the subject of magnetohydrodynamic waves in so far as it gives the

directional distribution of waves from a local source, a point completely missed by the plane wave approach.

S. D. Nigam, India

4447. Abonyi, I., Magnetohydrodynamic waves in a toroidal cavity resonator (in Hungarian), *Magyar Fiz. Foly*, **8**, 6, 463–469, 1960.

Theoretical solutions of the following boundary-value problem are presented: A toroidal cavity is filled with an ideal (inviscid, infinitely conductive) fluid. Small perturbation equations are assumed to be toroidal and of constant intensity across the circular cross section.

Three types of solutions were obtained. Purely longitudinal acoustic solutions, purely transversal Alfvén waves, and the mixed (slow and fast) waves. Since no curvature effects are taken into account the solutions are really those of an infinite cylinder merely restricted to periodic solutions along its axis. (The assumption of constant magnetic field across the cross section of a torus would require d c electric currents be present everywhere according to Maxwell's equations.) L. S. G. Kovasznay, USA

4448. Schumann, W. O., Hydromagnetic waves in plasmas (in German), *Z. Angew. Phys.* **11**, 7, 259–264, July 1959.

The title problem is investigated for the case when the gyrofrequencies of the ions and electrons greatly exceed the corresponding collision frequencies (ν) and also the wave frequency (ω). The calculation allows for electron diffusion. It is found that when $\omega \ll \nu$ this diffusion can be ignored, and then the phase velocity is independent of the direction of propagation. Relations are given for the phase velocity and the damping, and some special cases are discussed. L. C. Woods, England

4449. Hoh, F. C., and Lehnert, B., Diffusion processes in a plasma column in a longitudinal magnetic field, *Physics of Fluids* **3**, 4, 600–607, July/Aug. 1960.

The paper—an extension of earlier studies of one of the authors—deals with the behavior of lowly ionized, quasi-neutral plasmas, in longitudinal axisymmetric magnetic fields, generated by long magnetic coils ($L = 3.5$ m) wound around discharge tubes of small radii ($R = 0.543$, 0.70, and 1.00 cm), the inside pressure being varied between 0.1 and 4.0 mm Hg.

The results of the numerous experiments made by authors with rare gases, hydrogen and nitrogen are illustrated in several synoptic figures and discussed on behalf of an elementary theory, based upon the binary collision-diffusion theory of nonuniform gases in electromagnetic fields. The theory used proves to be satisfactory only up to a certain "critical" value B_c of the magnetic field, where the potential drop generally attains a minimum value. Contrary to this theory, for stronger magnetic fields the potential drop, and consequently the transverse diffusion rate of the charged particles, increases with the field, seeming to tend toward an asymptotic value; this behavior, called "abnormal" by authors, is considered as probably due to transverse, asymmetric electric fields caused by charge-density fluctuations, and possibly also by sound waves traversing the plasma during the discharge process.

The position of the critical or transition point is shown as depending on the nature and pressure of the gas, the discharge current, the ratio of the tube length to its radius, etc., and seems to respect a relation $B_c X R$ = constant.

L. G. Hamburger, Roumania

4450. Wright, J. P., Diffusion of charged particles across a magnetic field due to neutral particles, *Physics of Fluids* **3**, 4, 607–610, July/Aug. 1960.

The purpose of the paper is to evaluate the influence which neutral particles may exert on the diffusion of charged particles across a magnetic field. In order to simplify calculation, the field is considered uniform, the cyclotron frequency of the charged particles as largely exceeding their collision frequency, so that the corresponding velocity distribution functions become solutions of a stationary collision-free Boltzmann equation; for the neutral particles a Maxwellian velocity distribution is admitted; the number densities are assumed as varying in only one direction.

The flux due to the presence of non-charged particles and that arising from collisions between particles with opposite charges, as well as the ratio of these fluxes, are calculated for two alternatives, namely for an elastic scattering, the collision cross section being inversely proportional to the velocity square, and for a constant collision cross section. The solutions so obtained are applied to a mixture of neutrons and a neutral proton-electron gas. Under similar conditions it results that the ratio of the two fluxes is of the order of 10^3 in the first case, and of 10^2 in the second.

The importance of the flux caused by the presence of neutral particles depends on the energy range, the collision cross section, the neutron concentration, the density numbers of the charged particles, etc. L. G. Hamburger, Roumania

4451. Ludford, G. S. S., The effect of a very strong magnetic cross-field on steady motion through a slightly conducting fluid, *J. Fluid Mech.* **10**, 1, 141–155, Feb. 1961.

Paper studies theory of conducting fluid flow past body in two dimensions, in the reasonably realistic limit of high Hartmann number (inviscid), low magnetic Reynolds number (uniform field imposed perpendicular to free stream, unperturbed by fluid motion) and low velocity or low density, etc., such that inertia forces are negligible in comparison with magnetic forces. In the limit all quantities become invariant in the field direction and the solution is not unique. Author resolves ambiguity by re-introducing inertia and studying the behavior of the solution as inertia tends to zero. Conclusion is that flow ahead of the body is perturbed. The behavior as the limits of zero viscosity and zero magnetic Reynolds number are approached is not explored. Paper is a good cautionary example of the perils of extreme, limiting cases in magnetohydrodynamics. J. A. Shercliff, USA

4452. Hurley, J., Interaction of a streaming plasma with the magnetic field of a line current, *Physics of Fluids* **4**, 1, 109–111, Jan. 1961.

The interaction between the magnetic field of a line current and a rare plasma streaming toward it is investigated. It is assumed that the field of the line current is confined to a cavity with the plasma particles specularly reflected in a thin layer of the cavity wall. The shape of the cavity wall and the field in the cavity are calculated. From author's summary

4453. Hasimoto, H., Effect of a magnetic field on the flow of a conducting fluid through an orifice, *Physics of Fluids* **4**, 1, 161–162 (Letters to the Editor), Jan. 1961.

4454. Gupta, A. S., On the flow of an electrically conducting fluid near an accelerated plate in the presence of a magnetic field, *J. Phys. Soc. Japan* **15**, 10, 1894–1897, Oct. 1960.

The effect of a transverse field on the flow of fluid near a plate which moves with velocity proportional to t^n is investigated. The equation of motion is solved by Laplace transformation and the drag coefficient is computed in the case $n = 1$. The drag increases, as expected, as the field increases. H. K. Moffatt, England

4455. Drazin, P. G., Stability of a broken-line jet in a parallel magnetic field, *J. Math. Phys.* **39**, 1, 49–53, Apr. 1960.

Author considers two incompressible inviscid electrically conducting fluids, namely a jet of uniform velocity surrounded by an

infinite fluid at rest, with a uniform parallel magnetic field, and investigates the stability of the flow to small perturbations. The most interesting result is that at all finite magnetic Reynolds numbers the flow is unstable to long-wave disturbances. There appears to be a misprint on line 10 of page 53 where "stability" should read "instability". N. Curle, England

4456. Starr, W. L., and Kash, S. W., Experimental results with a collinear electrode plasma accelerator and a comparison with ion accelerators, *ARS J.* **31,** 1, 58–61, Jan. 1961.

A collinear electrode plasma accelerator using either plasma derived from an exploding wire or from electrode erosion has been in operation for some time. Maximum particle velocities approaching 10^7 cm per sec and impulses of 2×10^3 dyne · sec have been achieved. Effective specific impulses, computed from measured impulse and propellant mass, greater than 2000 sec have been obtained for the exploding wire measurements. For certain conditions of operation, the efficiency is greater than 30 per cent. Efficiencies of 50 per cent or more should be possible. The practicality of deriving the plasma by electrode erosion is demonstrated, and effective specific impulses of about 6000 sec are indicated. From authors' summary

4457. Vaughan-Williams, R. W., and Haas, F. A., An error in the thermal conductivity for a fully ionized gas, *Phys. Rev. Letters* **6,** 4, 165–167, Feb. 1961.

4458. Ziemer, R. W., Heat transfer gage for use in highly ionized gases, *ARS J.* **31,** 1, 78–80 (Tech. Notes), Jan. 1961.

4459. Ergun, A. N., Axisymmetric solutions of the incompressible magnetohydrodynamic equations, *Quart. J. Mech. Appl. Math.* **13,** 4, 408–427, Nov. 1960.

Paper investigates particular cases of a known general form of the solution of the (vorticity) equation for the Stokes stream-function. The effects of finite conductivity are included. One of the results proves to be a generalization of Hill's spherical vortex. The results obtained are not applied to any practical problems, and the paper lacks any account of their physical significance.
L. C. Woods, England

4460. Chekmarev, I. B., One-dimensional flow of a compressible gas in a pipe in the presence of a transverse magnetic field, *Appl. Math. Mech. (Prikl. Mat. Meckb.)* **24,** 2, 553–555, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Golitsyn and Stanuikovich ["Certain questions in magnetogasdynamics with allowance for the finite cross section of the strain," *Zh. Eksp. Teor. Fiz.* **33** 6 (12), p. 1417, 1957] have analyzed one-dimensional flow of a compressible gas in the presence of a transverse magnetic field. In the present paper the author carries out the integration of their system of equations for an inviscid, non-heat-conducting gas, with the boundary conditions (u_0, p_0, ρ_0, T_0, H_0) given at $X = X_0$. After some operations on the system of equations the author obtains an integral equation for u. Introducing two characteristic velocities, which are the roots of a characteristic equation, into the equation for the velocity furnishes a simpler formula (integral equation) for u. Taking logarithms and then differentiating, integrating again and making use of the boundary conditions for the velocity furnishes an implicit equation for u and X. Another formulation of the same equation is the expression for the rate du/dx. A brief discussion of the results and the derivation of the formula for the strength of the magnetic field close the paper.
M. Z. v. Krzywoblocki, USA

4461. Greenspan, H. P., Flat plate drag in magnetohydrodynamic flow, *Physics of Fluids* **3,** 4, 581–587, July/Aug. 1960.

The purpose of this paper is to elucidate the phenomena of sub-Alfvénic flow of a viscous fluid past a flat plate, which were pointed out in papers of the author and G. F. Carrier [*J. Fluid Mech.* **6,** 77–96, 1959 and **7,** 22–32, 1960; AMR **13** (1960), Revs. 3106 and 5448] and H. Hasimoto [*Physics of Fluids* **2,** p. 337, 1959]. This category of steady flows involves parallel velocity and magnetic fields at infinity and stream speed less than the Alfvén-wave speed. The same (Oseen) linearization is used as in the author's earlier paper. Here the solution is formulated by Fourier-integral methods. Approximate results are obtained for zero and for infinite conductivity when the Reynolds number *Re* based on plate length is large; these exhibit, in their leading terms, singularities at the leading and trailing edges, respectively. These conclusions are in agreement with Hasimoto's, which pertain to a general class of bodies. For arbitrary conductivity and large *Re* the skin friction has been calculated; the results show how the shear migrates from the front to the back portions of the plate as conductivity increases. The relative strengths of trailing- and leading-edge singularities indicates the relative strengths of the precursor and the wake. Finally, for small *Re* a different approximation is made and results are obtained which augment those of the Greenspan-Carrier 1959 paper listed above, where a small *Re* approximation was made for infinite conductivity.
W. R. Sears, USA

4462. Diprima, R. C., Some variational principles for problems in hydrodynamic and hydromagnetic stability, *Quart. Appl. Math.* **18,** 4, 375–385, Jan. 1961.

Variational methods are applied to three stability problems: (1) Taylor's problem of a viscous fluid between concentric cylinders rotating in the same direction, (2) the same problem with axial magnetic field, (3) the inhibition by a magnetic field of convection of a fluid heated from below. All problems reduce to that of the solution of linear differential equations of high order with constant coefficients. Author gives simple transformations to extend the method to non-self-adjoint problems. The solution is expanded in terms of orthogonal functions, whereby some boundary conditions are termwise satisfied, others enter as constraining conditions by way of Lagrange multipliers. Stationary value of the variational integral is found by partial differentiation with respect to series coefficients. This yields a transcendental equation for the critical stability parameter. A direct attack on the problem by means of series solutions without recourse to variational method is also illustrated, and it is pointed out that, while in this case all boundary conditions have to be accommodated explicitly, variational methods may contain some of them as "natural" conditions, i.e. they are automatically satisfied.
A theoretical paper. P. Savic, Canada

4463 Gershuni, G. Z., and Zhukhovitskii, E. M., Flow of a conducting fluid around a sphere in a strong magnetic field, *Soviet Phys.-Tech. Phys.* **5,** 8, 870–871, Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 8, 925–926, Aug. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4464. Pain, H. J., and Smy, P. R., Experiments on power generation from a moving plasma, *J. Fluid Mech.* **10,** 1, 51–64, Feb. 1961.

Electrical power of 0.32 MW has been extracted for a period of 100 μsec from a plasma of shock-ionized argon traveling at 4×10^6 cm per sec through a magnetic field of 10,000 G. Currents of more than 10,000 amp are drawn and the resulting modification of the applied field is considered. Maximum power is obtained when the external load is matched to the plasma generator which has an internal impedance equivalent to its own electrical resistance in series with a resistance arising from its behavior as a compressible fluid.

Values of the electrical conductivity of the plasma obtained in these experiments (about 3×10^3 mho per meter) show that the plasma resistance is controlled by electron motion and these values are in good agreement with those found by other methods.

From authors' summary by J. Rom-Rabinowicz, Israel

4465. Polovin, R. V., The motion of a piston in a conducting medium, *Soviet Phys.-JETP* **11,** 5, 1113-1120, Nov. 1960. (Translation of *Zh. Eksp. Teor. Fiz., SSSR* **38,** 5, 1544-1555, May 1960.

Author considers magnetohydrodynamic waves which arise when a piston moves in a perfectly conducting medium in the presence of a magnetic field. If the transverse velocity component of the piston exceeds the velocity of sound in the undisturbed medium, then a magnetic field is generated; in this case, the magnetic pressure becomes comparable with the hydrostatic pressure. At supersonic velocities, a vacuum is formed between the piston and the medium (cavitation). Compared with ordinary hydrodynamics, additional cases of cavitation appear when the piston moves with supersonic velocity in the direction perpendicular to the normal, and also when the piston moves in, if the angle between its velocity vector and the normal to its surface exceeds 70° (for an ideal gas with $\gamma = 5/3$). Increase of the piston velocity component perpendicular to the normal decreases the drag. When cavitation occurs, the drag is four times less than in the case of motion of the piston in the direction normal to its surface. From author's summary

4466. Rostoker, N., Kinetic equation with a constant magnetic field, *Physics of Fluids* **3,** 6, 922-927, Nov./Dec. 1960.

The collision operator is derived for the case of a spatially homogeneous plasma subject to a constant external magnetic field. A generalization of Lenard's method is employed.

From author's summary

4467. Fahleson, U. V., Experiments with plasma moving through neutral gas, *Physics of Fluids* **4,** 1, 123-127, Jan. 1961.

The behavior of a rotating plasma machine has been investigated. It is found that when the gas pressure and the current are varied within very wide limits the burning voltage remains constant. The voltage is proportional to the magnetic field. The reason for this behavior seems to be the existence of a critical velocity, above which a strong interaction between a plasma and a neutral gas takes place. From author's summary

4468. Pipkin, A. C., Electrical conductivity of partially ionized gases, *Physics of Fluids* **4,** 1, 154-158, Jan. 1961.

Transfer equations as given by Burgers are used to calculate the electrical conductivity of a partially ionized gas in the presence of a magnetic field. The result corresponds to what Chapman and Cowling call a "second approximation," taking into account the influence of heat transfer. The importantance of various terms occurring in the final expression is analyzed.

From author's summary

4469. Allen, M. A., and Kino, G. S., Interaction of an electron beam with a fully ionized plasma, *Phys. Rev. Letters* **6,** 4, 163-165, Feb. 1961.

4470. Marlotte, G., and Demetriades, A., Electrical discharges in hypersonic flows, *Physics of Fluids* **3,** 6, 1028-1029 (Letters to the Editor), Nov./Dec. 1960.

Aeroelasticity

(See Revs. 3980, 4130)

Aeronautics

(See Revs. 4207, 4323, 4347, 4416)

Astronautics

(See also Revs. 3985, 3986, 3991, 4416, 4417)

Book—4471. Ehricke, K. A., Space flight, Vol. 1: Environment and celestial mechanics, New York, D. Van Nostrand Company, Inc., 1960, xiii + 531 pp. $14.50.

The addition of Ehricke's three-volume book, "Space flight," to the Van Nostrand series known as Principles of Guided Missile Design is welcomed. The first volume of the trilogy, entitled "Environment and celestial mechanics", is the subject of the present review. Sequel volumes are "Dynamics" and "Space operations," to be published in August and December 1961.

The author is the recognized dean of preliminary design of the missile and space industry and accordingly his book intends to serve mainly advanced systems engineers who are responsible for conceptual designs and feasibility studies.

The first part of the book ("Concept of space flight and environment") is especially successful in serving this goal. The review of missile development, the utility aspects of space flight, the scientific aspects and applications of instrumented satellites, the detailed discussion of lunar and interplanetary probes and of manned operations are masterfully handled. The last chapter of the first part treats the solar system environment, gives a short course in its descriptive astronomy and discusses the radiation and particle "climate." The concisely tabulated presentation of planetary, satellite, asteroid, etc., data will appeal to the engineer.

In the second part, author's attempt to treat applied celestial mechanics is only slightly less successful than his enjoyable and useful treatment of environment. This probably is due more to the subject matter than to the author's ability. This part is presented in three major chapters: "Central force field," "Orbit in space," and "Perturbation analysis." The first subject is treated with unusually systematic thoroughness—the chapter is concluded with a comprehensive collection of formulas related to the two-body problem, arranged alphabetically in terms of 26 parameters. The chapters on orbits contain the geometries of the various coordinate systems, transformations and a sketchy description of orbit determination techniques. A laudatory comment is justified for the geometry and this reviewer is inclined to agree with the author regarding orbit determination techniques in recommending Professor P. Herget's excellent (but almost unavailable) book on this subject, The third subject, perturbation analysis, might disappoint preliminary designers performing design and feasibility studies when quick information or simple techniques are needed in this area. As author clearly states, there is no "easy" way of mastering perturbation theory and this reviewer notes with interest that this superbly written bssically engineering book devotes to perturbation theories 100 pages of a total of 240 pages dedicated to celestial mechanics.

The basic problem which the systems engineers and advance preliminary designers face in the area of trajectory analysis is simply that there are no "exact" or approximate formulas describing the paths of bodies in gravity fields under the influence of more than one moving force center. The two-body (or central force field) problem is trivial; analytical expressions are well known and they describe the motion of a body (space vehicle) in the gravity field of another body (e. g. homogeneous spherical planet). If another (third) body is introduced into the field (e. g. Earth + Moon + space vehicle), the description of path at present does not exist in "closed form" and the advance designer therefore is forced to look for approximate formulas with known accuracies. To "ap-

proximate" a three-body problem with two-body solutions and to "patch" several conic sections is often meaningless and might lead to erroneous results. Presently existing general perturbation techniques are useful for astronomical purposes and their application to satellite motion problems in the gravitational field of the Earth have been eminently demonstrated. Essentially n-body ($n > 2$) fields and problems of interest in space vehicle trajectories are at the present time not tractable to simplified analysis nor are "useful engineering equations" available. Because of this, preliminary designers and the authors of books, even of as excellent a book as this, find it rather difficult to create interest in general perturbation techniques.

A large number of well-selected and properly grouped references are very helpful. The "problems" attached to some of the chapters are of more technical interest than of instructional value. Reviewer does not think the book needs recommendation, since it is already known and widely used among engineering students. The last word is used advisedly since "we are all students as far as space is concerned," to quote the author.

V. G. Szebehely, USA

4472. Musen, P., A modified Hansen's theory as applied to the motion of artificial satellites, NASA TN D-492, 39 pp., Nov. 1960.

Paper deals with perturbed motion of artificial satellites, based upon Hansen's treatment. After selecting an auxiliary elliptic path in accordance with Kepler's law, the real positions of the satellite are determined as deviations from the auxiliary path. The paper deals with perturbations in the orbit plane and of the orbit plane; author modifies Hansen's treatment by application of vectorial representation on the one hand, and by adaptation of the problems for numerical solution by means of computers on the other hand. The paper has the typical texture of a "research-project", giving laborious and involved results and lacking some essential novelty of approach.

J. Mandelker, USA

4473. Batrakov, Yu. V., Determination of the preliminary orbits of artificial satellites from observations with time approximately known, *ARS J.* **30,** 9, 859-864 (Russian Suppl.), Sept. 1960.

The orbit of a satellite can be computed if three observations yield the coordinates of the satellite on the celestial sphere and the instants of observation accurately. Author shows how to compute the orbit from four observations if the time is measured accurately for two observations only. A numerical example is given.

P. M. Treuenfels, USA

4474. Kulikov, D. K., and Batrakov, Yu. V., Method for improving orbits of artificial satellites of the earth using observations with approximate values of time, *ARS J.* **30,** 9, 865-874 (Russian Suppl.), 1960.

A preliminary orbit of an earth satellite having been determined (see preceding review), authors now describe a method of data reduction to improve the elements of the orbit. The use of satellites for geodetic purposes is discussed. A numerical example is given.

P. M. Treuenfels, USA

4475. Kulakowski, L. J., and Stancil, R. T., Rocket boost trajectories for maximum burnout velocity, *ARS J.* **30,** 7, 612-618, July 1960.

The problem considered is the definition of optimal trajectory programs which allow a given rocket boost vehicle to launch its payload with maximum velocity, for prescribed burnout values of the path angle and altitude. Methods of the calculus of variations are employed to achieve the desired solutions. The equations of motion consider a spherical nonrotating Earth model and include the effects of aerodynamic forces. Inequality constraints suitable for satisfying certain practical vehicle design and physiological limits are introduced. An optimal coasting period of initially unspecified duration may be included in the trajectory to provide increased burnout altitude. Numerical results illustrating the utility and validity of the method are included.

From authors' summary by W. W. Berning, USA

4476. Graham, E. W., A class of optimum trajectory problems in gravitational fields, *J. Aero/Space Sci.* **27,** 4, 296-303, Apr. 1960.

The problem considered is that of the transfer of a body from one set of coordinates (space and velocity) to another with minimum impulsive effort (fuel consumption). The idealized problem with no resistance is discussed by using coordinate axes which follow a coasting trajectory but are nonrotating. This simplifies the kinematic treatment. The previously known solution that the optimum is obtained by impulses only at the ends of the path for a constant gravitational field is extended to centrally directed fields. The method is possibly capable of extention. It is exact, with the conditions postulated and not an approximation.

R. C. Knight, England

4477. Batrakov, Yu. V., and Proskurin, V. F., Perturbations of orbits of artificial satellites due to air resistance, NASA TT F-46, 13 pp., Nov. 1960.

Presentation of a general form of the first-order perturbations in the elements of an elliptic satellite orbit, caused by the resistance of the air, is undertaken. It was assumed that the earth's atmosphere has a fully spherical density distribution and that the attraction of the earth can be replaced by the attraction of the material point, placed at its center of inertia and having the same mass as the earth. These assumptions, along with the secular perturbations, are used to obtain faster perturbations whose periods do not exceed the period of one rotation of the satellite.

From authors' summary

4478. Townsend, G. E., Kork, J., and Kraft, J. D., Nomograms for the solution of orbital parameters, *Aerospace Engng.* **19,** 12, 12-13, Dec. 1960.

4479. Eggleston, J. M., Optimum time to rendezvous, *ARS J.* **30,** 11, 1089-1091 (Tech. Notes), Nov. 1960.

4480. Ewart, D. G., On the motion of a particle about an oblate spheroid: Part 2, Calculation of the time of flight, *J. Brit. Interplanetary Soc.* **17,** 11, 393-397, Sept./Oct. 1960.

In a previous paper [D. G. Ewart, title source **17,** p. 162, 1959-1960] the results of a solution of the equations of motion of a particle about an oblate spheroid to the first order of perturbations, using the method of variation of elements, were presented.

Numerical applications of the theory indicated the presence of an error in the time-of-flight equation. Three possible sources of the error in the theory are considered. The error is found to arise from the omission of a term in the equation for the rate of change of the mean longitude of the epoch. A theoretical expression for the correction required by the time of flight is derived.

From author's summary by D. C. Leigh, USA

4481. Allen, W. A., Effect on a rocket of the oblateness of a planet, *ARS J.* **30,** 7, 623-627, July 1960.

Perturbing effects of oblateness of a planet on satellite orbits have been studied extensively and reported in the literature. Author applied effects of oblateness to trajectory of a rocket launched from or near the surface of a planet and solves equation of motion for flight along a radial vector from center of planet subject to restriction that sum of radial acceleration and acceleration of gravity remains constant. Specifically, second- and fourth-order corrections to radial acceleration, radial velocity, and time of

flight (relative to zeroth-order calculation with planet considered a homogeneous sphere) are considered.

Author includes example of flight from planet earth and shows that oblateness effects yield a maximum acceleration correction of 0.031 m/sec^2 (earth's surface), velocity correction of 1.95 m/sec (approximately 1.4 earth radii), and a maximum time error of 60 milliseconds at infinity. Author concludes that results from the restricted problem suggest that corrections due to oblateness of a planet are generally small or negligible.

W. W. Berning, USA

4482. Carraway, A. B., Edwards. F. G., and Keating, Jean C., Investigation of the static stability characteristics of two stages of a three-stage missile at a Mach number of 4.00, NASA TN D-651, 18 pp., Mar. 1961.

An investigation has been conducted in the Langley Unitary Plan wind tunnel to determine the aerodynamic characteristics of two stages of the 1/7-scale Trailblazer II configuration. Included in this investigation was the determination of the effect of fin cant, fin size, and length of stage coupling. Tests were performed at a Mach number of 4.00 and a Reynolds number of 13.26×10^6 based on model length. The angle-of-attack range was from about $-5°$ to $5°$ and angle of sideslip was varied from approximately $-4°$ to $4°$.

The results showed that the configuration is longitudinally and directionally stable with either of the two sets of fins. Canting each of the vertical fins $2°$ leads to an incremental rolling-moment coefficient of about 0.30 to 0.35 at angles of attack from $-4°$ to $4°$. Increasing stage-coupling length leads to decreases in both longitudinal and lateral stability; however, if the center of gravity of the configuration is moved forward with this change, it is believed there would be little or no effect of the variable on the stability characteristics of the configuration.

From authors' summary

4483. Vakhnin, V. M., and Beletskiy, V. V., Observation of an artificial satellite by the expectation method, NASA TT F-48, 10 pp., Nov. 1960.

An expectation method is presented for predicting the appearance of an artificial satellite within the observation range of the spotting equipment. For successful application of the method, it is important to take into account only one variable factor, namely, the rate of orbit precession. From authors' summary

4484. Kelley, O. A., Jr., Parametric weight study of a manned space entry vehicle, *Aero/Space Engng.* **19,** 10, 40-49, 77, Oct. 1960.

Paper describes a manned space entry vehicle weight study which is an integral part of a comprehensive configuration investigation. The purpose of this study is to show weight comparison or trend data between different vehicles and heat-protection schemes. Both lifting and ballistic-type entry vehicles are investigated for satellite and circumlunar missions. The structural cooling methods studied are radiation, ablation, and a combination of radiation plus ablation. The method or approach used for this study is presented in detail with sample calculations included.

From author's summary

4485. Charwat, A. F., Possible aerodynamic lift and moment at the perigees of low altitude satellites, *Physics of Fluids* **3,** 5, 829-830 (Letters to the Editor), Sept./Oct. 1960.

4486. Newton, R. R., Stabilizing a spherical satellite by radiation pressure, *ARS J.* **30,** 12, 1175-1177 (Tech. Notes) Dec. 1960.

4487. Vaughan, V. L., Jr., Landing characteristics and flotation properties of a reentry capsule, NASA TN D-653, 36 pp., Feb. 1961.

A conical capsule model with a segment of a sphere for an impact surface was tested to determine rigid-body impact accelerations on sand and on water for a variety of flight paths and contact attitudes. Accelerations were measured along the X-axis (roll), Z-axis (yaw), and at positions in front and in back of the X-axis. Further tests were made to determine the flotation properties of the same model configuration.

From author's summary

4488. Rodriguez, D., Meteoroid shielding for space vehicles, *Aerospace Engng.* **19,** 12, 20-23, Dec. 1960.

Ballistics, Explosions

(See also Revs. 4153, 4154, 4161, 4402, 4444, 4472, 4476)

4489. Glass, I. I., Aerodynamics of blasts, Univ. Toronto, Inst. Aerophys. Rev. 17, 35 pp. + figs., Sept. 1960.

For the past decade theoretical and experimental investigations have been conducted at the Institute of Aerophysics on shock-tube flows. Recently, this work was extended to include the analogous flows generated by spherical and cylindrical explosions. Glass spheres and cylinders are pressurized by means of compressed gases or combustible mixtures and are shattered to generate an explosion. Similar methods can be used for implosions, underwater explosions, and wave interactions. The finite mass, strength, and breaking time of the glass diaphragms impose some limitations on certain experiments where this technique is used to generate a blast wave. Nevertheless, the method has proved very valuable in the study of many basic properties of spherical and cylindrical blast phenomena that have been investigated using piezo-pressure gauges and several schlieren and shadowgraph techniques. Some consideration is given to intense explosions from concentrated energy sources for spherical, cylindrical, and planar blasts, and explosions and implosions generated from finite sources with the same geometries. Blast-wave simulators for aerodynamic tests and the dynamic testing of structural components are briefly discussed. From author's summary by C. Franze, Germany

4490. Ericsson, U., and Edin, K., On complete blast scaling, *Physics of Fluids* **3,** 6, 893-895, Nov./Dec. 1960.

Experimental evidence is presented for the ability of Sachs's complete energy scaling to account for the influence of ambient pressure and temperature at not too small distances from the charge. From authors' summary

4491. Voitsekhovskii, B. V., and Kotov, B. E., Optical investigations of the front of a following detonation wave (in Russian), *Izv. Sibirsk. Otd. Akad. Nauk SSR* no. 4, 74-80, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 3571.

Photographs of gaseous detonations have been obtained by a compensation method consisting of exposures made on a film moving with the speed and in the direction of the coordinate system enveloping the image of the recorded phenomenon. It is shown that the greatest brightness is exhibited by the front of the transverse detonation wave. This result contradicts the theory of lagging detonation developed by Ia. B. Zeldovich, according to which the most intensive reaction takes place at the discontinuity of the front in advance of the traveling shock wave. The authors demonstrate that the shock wave compresses the cold gas, causing it to become heated, while the reaction develops in the transverse detonation wave, propagated through the compressed gas. Utilizing

the property of gelatine, first observed in the course of the present research, of changing its transparency for light rays according to the magnitude of the pressure in the shock wave, the authors have obtained photographs, illustrating the distribution of the pressure fronts on the surface of the (explosion) tube. The photographs clearly show that the transverse detonation wave is more powerful than the shock wave moving in advance of the detonation wave front.

A. N. Dremin

Courtesy Referativnyi Zhurnal, USSR

4492. Anisimov, S. I., Powerful explosion in a nonideal gas, *Soviet Phys.-Tech. Phys.* **5,** 9, 1057–1061, Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 9, 1124–1127, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4493. Kovitz, A. A., Some comments on standing detonation waves in a combustion tunnel, *J. Aerospace Sci.* **28,** 1, 75–76 (Readers' Forum), Jan. 1961.

Acoustics

(See also Revs. 4147, 4323)

4494. Tyutekin, V. V., Diffraction of a plane sound wave by an infinite cylindrical cavity in an elastic medium with an arbitrary angle of incidence, *Soviet Phys.-Acoustics* **6,** 1, 97–102, July/Sept. 1960. (Translation of *Akust. Zh., USSR* **6,** 1, 101–106, Jan./Mar. 1960 by Amer. Inst. Phys., New York, N. Y.)

Diffraction of a plane sound wave by an infinite cylindrical cavity in an isotropic elastic medium for arbitrary angle of incidence with respect to the axis of the cavity is considered. The incident wave is expanded in cylindrical waves. The displacement of points in the medium can be written as grad φ + rot π where $\varphi = \varphi_i + \varphi_r$ is the total scalar potential of the incident and reflected waves and where Π is the vector potential of the reflected wave. Solutions are obtained for φ and for Π in the form of sums of cylindrical waves of various orders. The coefficients in these sums are ratios of determinants whose elements involve Hankel functions and their derivatives. The dependence of the amplitude of the zero wave on the angle of incidence and the frequency of the incident wave is discussed in detail.

N. D. Kazarionoff, USSR

4495. McSkimin, H. J., Empirical study of the effect of diffraction on velocity of propagation of high-frequency ultrasonic waves, *J. Acoust. Soc. Amer.* **32,** 11, 1401–1404, Nov. 1960.

The effect of diffraction in increasing the velocity of propagation over the plane wave value is determined experimentally by phase comparison of reflected longitudinal waves in fused silica blocks. It is shown that the parameters of transducer diameter, path length traveled, and frequency can usually be adjusted to keep the fractional excess velocity under 0.02% for measurements in the 20–100-Mc range. This consideration is of particular importance in connection with precision measurement of elastic moduli.

From author's summary

4496. Ford, G. W., and Meecham, W. C., Scattering of sound by isotropic turbulence of large Reynolds number, *J. Acoust. Soc. Amer.* **32,** 12, 1668–1672, Dec. 1960.

Theory is developed for the scattering of monochromatic plane acoustic waves on an incompressible isotropic turbulent velocity field having an important "inertial subrange." The scattering formulas are derived along well-established lines on the basis of single scattering and infinitesimal sound wave amplitude. Since for the computation of the retarded potential one needs to know the space-time correlations of the turbulent field, authors make the assumption that these space-time correlations can be approximated by using a sort of Taylor's hypothesis and using the small eddies for the scattering effect, but calculating the resulting Doppler effect from the large-scale eddy motion. It is regrettable that authors did not make the easy extension to the scattering of sound on turbulent density (entropy) fluctuations. Authors do not refer to published space-time correlation measurements [A. Favre et al, Quelques Mesures de Correlation dans le Temps et l'Espace en Soufflerie, *Recb. Aero.* no. 32, p. 21, Mar.–Apr. 1953; Correlation dans le Temps et l'Espace, avec Filtre de Bande en Aval d'une grille de Turbulence, *Recb. Aero.* no. 40, p. 7, July–Aug. 1954; (English translation) Apparatus for measurement of time and space correlation, NACA Tech. Memo no. 1371, Apr. 1955; see AMR **7**(1954), Rev. 281; **8**(1955), Rev. 3135] that would have permitted a check on their assumption.

L. S. G. Kovasznay, USA

4497. Mechel, Fr., Sound attenuation and sound amplification in airflow through ducts with absorbing walls (in German), *Acustica* **10,** 3, 133–148, 1960.

Noise absorption in airflow through ducts lines with porous absorbing materials is investigated at flow velocities up to 80 m/sec and in the frequency range of 200–3000 c/s. Noise amplification may occur in ducts with periodically changing cross sections. This effect is discussed and compared with electric amplification in traveling wave tubes.

F. E. Borgnis, Switzerland

4498. Brosio, E., Measurement of the sound insulation by random and by normal incidence of sound, *Acustica* **10,** 3, 173–175, 1960.

Authors compare experimental data of transmission loss measured by random and by normal incidence of sound. The theory of London is found to agree well with these data: for given partitions it is then possible to calculate one of the two values when the other has been determined experimentally.

From author's summary

4499. Brooks, R., Determination of the velocity of sound in distilled water, *J. Acoust. Soc. Amer.* **32,** 11, 1422–1425, Nov. 1960.

The velocity of sound is determined in distilled water over the temperature range from 22 to 25°C. The method used consists of measuring the transit time of two sound paths of different lengths traveling between the crystal faces of two barium-titanate transducers. With an accurate measure of the length, the velocity is determined from the quotient of $\Delta l/\Delta t$. It is believed that the accuracy of the measurements is within ±1 ft/sec.

From author's summary

4500. Schaaffs, W., Connection between the Guldberg law and temperature acoustic field in liquids (in German), *Acustica* **10,** 3, 160–166, 1960.

The velocity of sound decreases in direct proportion to temperature in most liquids and melts. With the help of the work hypothesis, that a linear extrapolation above the boiling point strikes the parabolic curve of the gas phase at a point corresponding to the critical temperature, a formula is deduced according to which the temperature-coefficient of the velocity in the liquid is proportional to that at the boiling point and inversely as the absolute value of the boiling temperature. The factor of proportionality is calculated for organic liquids, condensed gases, molten metals and alkali salts. The factor is closely related to the quotient of the Guldberg rule.

From author's summary

4501. Auberger, M., and Rinehart, J. S., Ultrasonic attenuation of longitudinal waves in plastics, *J. Appl. Phys.* **32**, 2, 219–222, Feb. 1961.

"Hughes' pulse technique for measuring longitudinal velocities has been adapted and extended to measure attenuation of longitudinal waves.... Data for attenuation.... in six different plastics (plexiglass, polystyrene, nylon 101, Formica XXN, polyethylene, and Teflon) are given for eight different frequencies ranging from 250 to 1000 kc. The results in plexiglass and polystyrene are compared with results obtained previously by other methods. Attenuation in nepers per wavelength has been found to decrease when frequency increases for all plastics, except for Teflon, which shows a well-defined peak at about 700 kc." (from authors' summary)

Longitudinal velocities are also given, with an estimated accuracy of ±2 per cent. Velocity is independent of frequency except for Teflon, in which it varies by ±3.5 per cent. The measurements were carried out at room temperature.

C. E. Chase, USA

Micromeritics

(See Revs. 4377, 4421, 4520)

Porous Media

(See also Revs. 4171, 4389)

4502. Rumiantsev, V. V., The stability of the rotational motions of a solid body with a liquid cavity, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 6, 1512–1524, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Rumiantsev considers the problem of rotational stability of a body (two moments of inertia equal) containing a cavity (rotational symmetry; axis of symmetry coincides with the principal axis of the inertia tensor of the solid which corresponds to the moment of inertia that is distinct) which is only partly filled with liquid. The author confines his attention chiefly to the stability of motion of the solid body; the question of the stability of the motion the liquid exerts an effect on the stability of motion of the body as a whole. Thus, he puts the question of the stability of the motion of the system relative to all variables which characterize the motion of the solid and of the liquid. In such a setting the problem of the stability of motion of the solid body, and of the liquid contained in its interior, leads to the investigation of the conditional stability of the system, that is, the stability relative to certain ones of the variables, and not to all of them that determine the motion of the mechanical system with an infinite number of variables. This problem is solved in the paper with the aid of a method of Lyapunov and by starting out with the complete equations of the perturbed motion.

From author's summary by A. E. Scheidegger, Canada

4503. Savinov, S. F., Uplift in concrete dam considering the permeability of concrete (in Russian), *Gidrotekh. Stroit.* **28**, 4, 40–42, Apr. 1959.

Gravity-type dam (triangular profile) made of concrete with permeability coefficient k_1 rests on medium with permeability coefficient k_0. Depth of pervious medium is limited to $2b$ (b = width of dam base). Headwater depth is H; tailwater depth is taken as $b = 0$, and $b = 0.25\,H$, respectively.

Author examines two cases: in case I a drainage gallery assumes various locations within dam profile; in case II a drain of width a laid on the foundation takes three different positions along the base line. The other variable is the relative permeability $c = {}^{k}1/k_0$.

By means of the "integrator EGDA – 6/53" author determines experimentally uplift F: $F = \frac{1}{2}\gamma(H-b)b\lambda\delta^{-1}$. λ accounts for nonlinearity of pressure distribution over base b (no gallery, no drain). δ expresses the influence of drainage gallery and of drain, respectively. Results are presented in tabular form for: (a) λ in range, $0 \leqslant c \leqslant 30$; (b) δ for case I in range $0.02 \leqslant c \leqslant \infty$, for 12 locations of gallery; (c) δ for case II in same range, for 4 values of a.

In case I, δ increases with c, and there seems to be a "most favorable" location of gallery. In case II, δ increases with a, and best position of drain is upstream.

Details of procedure and theoretical basis are not mentioned in paper.

G. H. Beguin, Switzerland

4504. Nikolaevskii, V. N., Convective diffusion in porous media, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 6, 1492–1503, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

Based upon the assumption of local isotropy of mixing, Nikolaevskii shows that the tensor D_{ik} in the dispersion equation of flow through a porous medium

$$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial x_i} D_{ik} \frac{\partial \Psi}{\partial x_k} - \bar{u}_k \frac{\partial \Psi}{\partial x_k}$$

(Ψ tracer concentration, t time, x_k spatial coördinate, $\bar{u}_k$ pore velicity) must have the form

$$D_{ij} = Q_{ijkl}\, \bar{u}_k\, \bar{u}_l / |\bar{u}|$$

where Q_{ijkl} is built up of only two independent constants. The latter are calculated for the system of principal axes of D_{ij}. Several solutions of the dispersion equation (dispersion of a tracer front) are given. As noted, the paper considers isotropic porous media only.

A. E. Scheidegger, Canada

4505. Barenblatt, G. I., and Zheltov, Yu. P., Fundamental equations of filtration of homogeneous liquids in fissured rocks, *Soviet Phys.-Doklady* **5**, 3, 522–525, Nov./Dec. 1960. (Translation of *Dokladi Akad. Nauk SSSR* (*N.S.*) **132**, 3, 545–548, June 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4506. Voronkova, L. D., Some problems of electrical simulation and the hydrodynamic analysis of the conjoint action of strata (in Russian), *Trudi Vses. Neftegaz. Nauk-i. In-ta* no. 12, 242–251, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4042.

A derivation is given for the equation of continuity of motion of a liquid in homogeneous petroleum beds of identical thickness and permeability, separated by less-permeable bridges. The equation is set up for the case of the conjugate action of two producing strata, after which the results are generalized for the case of conjugate action of n producing strata, on the condition that the roof of the first stratum, and the floor (sole) of the last, are absolutely inpermeable. As is usual, the motion (flow) is assumed to be two-dimensional (in one plane), steady, and following Darcy's law of percolation; the liquid is assumed to be homogeneous and incompressible; and the porous medium to be undistortable. In addition, the vertical component of the velocity of percolation through the strata is neglected as sufficiently small. Proof is given that, on the assumptions made, the analysis of a producing bed or stratum consisting of n layers, can be replaced by the analysis of a single, "equivalent" stratum, the parameters whereof will be determined by the expressions:

$$\left(\frac{k\,b}{\mu}\right)_x = \sum_{i=1}^{n} \frac{k_i b_i}{\mu_i} \ldots\ldots \qquad [1];$$

$$P_x = \left(\sum_{i=1}^{n} \frac{k_i b_i}{\mu_i} P_i\right) \Big/ \left(\frac{k b}{\mu}\right)_x \quad \ldots\ldots \quad [2];$$

wherein k_i, b_i, μ_i, P_i are, respectively; the coefficient of permeability, the thickness (of the stratum), the viscosity of the liquid, and the bed pressure on the i-th stratum ($i = 1, 2, \ldots, n$). On the condition that $(k\ b/\mu)_x = \text{const}$, the bed pressure P_x satisfies the Laplace equation. The results obtained are utilized to simplify the method of electrical simulation of conjugately-acting productive strata, and have been verified by an electrical integrating computer by modelling n conjugately acting strata with overflows between, and transforming them for the case of the "equivalent" stratum. A concrete example is analyzed.

P. F. Fil'chakov

Courtesy Referativnyi Zhurnal, USSR

4507. Khein, A. L., The calculation of the borehole pressures in a circular ring of holes incompletely opening the stratum in the presence of linear law percolation (in Russian), *Trudi Vses. Neftegaz. Nauk-i. In-ta* no. 10, 40–52, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4046.

On the assumption that a horizontal bed with impermeable roof and floor is drained by one or more imperfect bores, an approximate solution is given for the determination of the pressure required at the head of the bore to maintain the outflow (or inflow) of liquid (or gas) at a predetermined, constant rate. The stratum is assumed to be isotropic, the liquid compressible and of a constant viscosity, and the rate of percolation (filtration) of the liquid to follow Darcy's linear law. For the solution of the problem of the potential, author uses his hypothesis of invariability of the mean, weighted potential [A. L. Khein, title source no. 8, 1956]. The solution for the case of a single borehole is reduced to integrating the equation of heat conductivity for the density of the liquid in the case that a liquid is percolating, and for the Leibensohn function in the case that the percolating fluid is a gas. The equation is integrated by the sink method. It is demonstrated that the decrease in the potential at the surface of the borehole can be imagined as consisting of three parts: (1) The fall of the potential in an unsteady inflow to a perfect borehole; (2) the component representing the imperfection of the borehole in a steady inflow; (3) the part incorporating the time factor and depending on the nature of the imperfection of the borehole, representing a quantitative measure of the degree of deviation of the geometrical proportions of an imperfect, unsteady flow, from those of an imperfect, unsteady flow in the vicinity of the borehole. For the case of a circular battery of imperfect boreholes, an equation for the potential at the head of an imperfect bore in a circular battery is obtained, representing the sum of the work done by a particular bore, and the potential due to the work of the remaining bores of the battery.

V. K. Kuz'mina

Courtesy Referativnyi Zhurnal, USSR

4508. Trebin, F. A., and Shcherbakov, G. V., A simplified method for evaluating the results of restoration of pressure in boreholes, allowing for the inflow of liquid after stopping the bore (in Russian), *Neft. Kb-vo* no. 5, 37–41, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4049.

The possibility is investigated of applying parallel measurements of the pressure drop (by subsurface, buffer and casing pressure gauges), as suggested by A. P. Krylov, to the determination of a correction constant representing the influence of liquid inflow after the borehole has been stopped, on the initial segment of the curve of restoration of pressure:

$$z = \Delta P\ (t)/\Delta P\ (t)$$

where $\Delta P\ (t)$ is the actual pressure-restoration curve as measured by a subsurface pressure gauge, and representing the noninstantaneous closing of the bore; $\Delta P\ (t)$ is the referred curve of pressure-restoration coinciding with the asymptote to the real curve. An equation is written determining Z from the readings of the aforesaid pressure gauges, mean specific gravity of the liquid in the well, value of the undisturbed inflow, and the cross-sectional areas of the annular gap and the gusher pipes. Two specimen calculations are given, illustrating the simplicity and sufficient accuracy of the method. Its suitability for deep-pumping wells is pointed out.

V. N. Nikolaevskii

Courtesy Referativnyi Zhurnal, USSR

4509. Kozan, L. G., The hydrodynamic analysis of the elastic (expansible) state in the design of workings for petroleum production (in Russian), *Trudi Vses. Neftegaz. Nauk.-i. In-ta* no. 2, 292–313, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4047.

An account of the equipment and methods used in the practical calculation of the working of boreholes with given yields in a petroleum field, with an expansible state of the bed or stratum, using a petroleum slide rule developed by the author on the principle of a logarithm's scale. The petroleum slide rule mechanizes the process of calculating the pressure drop by the formula for point sources distributed on the circumference of a circle of radius r, and simplifies the numerical calculations for the elastic state, required in practical exploitation works. The rule has a number of different scales for distance, time, coefficient of piezo-conductivity, product of percolation rate by distance. All dimensional values are in practical units. The scale of values is: for distances from 1 cm to 100 km; for time, from 7 hours to 10,000 years; for the coefficient of piezo-conductivity, from 10^2 to $10^5 cm^2/sec$. A number of exemplary calculations are given: finding the pressure-drop (depression) at a point in the stratum with only one borehole working with a constant delivery, made steady by a jump; for a number of working boreholes; for the working of an imperfect borehole; and for the finding of the vector of the percolation velocity in these cases. It is pointed out that, in calculations for the case of varying delivery of the borehole, the delivery curve must be approximated by a stepped broken line, and assumed that when the delivery jumps from one step to another, a new borehole commences discharge at the value of the jump, this new bore being located at the same point in the stratum as the "old" borehole.

V. N. Nikolaevskii

Courtesy Referativnyi Zhurnal, USSR

4510. Koogin, L. G., The hydrodynamic analysis of strata working under an elastic (expansive) water head by the equations of the expansive and hydraulic head states (in Russian), *Trudi Vses. Neftegaz. Nauk-i. In-ta* no. 12, 314–322, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4048.

A petroleum pool is investigated, the feeding contour of which is represented by a circle of known radius and which is tapped by a group of bores located at its center. Account is taken of the elasticity of the stratum and the saturating liquid. The wells work with a constant delivery. Up to a particular time instant t_1, while the influence of the feeding contour on the redistribution of pressures inside the stratum is not yet apparent, the author proposes to conduct the hydrodynamic analysis by the expressions for an infinite stratum. After the time instant t_2, when the pressure in all parts of the stratum has become steady, the analysis proceeds by the equations for a steady state. Using a "hydrodynamic slide rule" constructed by the author, an approximate method is set up for determining the time instants t_1 and t_2, all calculations are speeded up, and an approximate solution for the pressure in the stratum is developed for the case that $t_1 < t < t_2$.

I. G. Gorokhova

Courtesy Referativnyi Zhurnal, USSR

4511. Gorokhova, I. G., An investigation of the development of the discharge volume of a liquid in an elementary percolation flow in the conditions of the elastic (expansive) state (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Neft' i Gaz.* no. 4, 61–66, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4050.

Numerical examples are presented for the calculation of the percolation of a liquid by the known expressions of the elastic state, for a single bore (well) in a continuous stratum and a well in the center of a stratum circularly bounded by a feeding contour or an impermeable partition. For the three calculated examples, charts are plotted for the change in the nondimensional delivery of the liquid through different cross sections and a tabular comparison is made between the quantities of liquid flowing in continuous and restricted strata. A part of the text is devoted to determining the inversion points of the curves of nondimensional delivery (flow volume) in relation to time and the cross-sectional radius, and the magnitude of the minimum flow volume to be considered.

This latter section lacks context with the preceding and clarity of presentation. There are inaccuracies and misprints.

L. G. Kogan

Courtesy Referativnyi Zhurnal, USSR

4512. Bouwer, H., Variable heat technique for seepage meters, *Proc. Amer. Soc. Civ. Engrs.* **87**, IR 1 (*J. Irrig. Drain. Div.*), 31–44, Mar. 1961.

A technique for measuring canal or reservoir seepage losses with seepage meters is proposed. The method also furnishes information regarding hydraulic conductivity of bottom material and seepage gradients. A resistance network analog was used for analyses of pertinent factors and flow components, and for obtaining examples to illustrate the method.

From author's summary

4513. Ogandzhanyants, V. G., and Egorova, I. I., Stability of water-petroleum contact in laminar porous media, *Soviet Phys.-Doklady* **5**, 5, 973–975, Mar./Apr. 1961. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **134**, 1, 59–61, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4514. Tukaev, A. G., The problem of determining the pressure function in strata of variable capacity under elastic conditions, *Soviet Phys.-Doklady* **5**, 5, 976–978, Mar./Apr. 1961. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **134**, 6, 1317–1319, Oct. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4515. Regirer, S. A., On the approximate theory of the flow of a viscous incompressible liquid in a tube with permeable walls, *Soviet Phys.-Tech. Phys.* **5**, 6, 602–605, Dec. 1960. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30**, 6, 639–643, June 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4516. Codreanu, D., Determination of pressure build up characteristics from pressure measurements, without shutting in the well (in Roumanian), *Petrol si Gaze* **10**, 4, 140–148, 1959.

Geophysics, Hydrology, Oceanography, Meteorology

(See also Revs. 4025, 4328, 4329, 4330)

4517. Vuorelainen, O., Thermal conditions in the ground from the viewpoint of foundation work, heating and plumbing installations and draining, *Acta Polytech. Scandinavica* no. 277, (Civil Engng. and Building Construction Series no. 7), 40 pp., 1960.

This pamphlet is essentially a review of the temperature distribution and heat flux in soils in cold areas. The effect of snow cover in reducing depth of freezing is discussed and curves presented. Equations for temperature, heat flux, and heat storage are given. The effect of the latent heat of freezing water is not included. The author uses the term "temperature conductivity" rather than the conventional "thermal diffusivity". The title of Figure 1 should read "Temperature conductivity" rather than "Thermal conductivity".

The pamphlet should be very useful to those concerned with underground thermal problems in cold areas.

R. V. Dunkle, Australia

4518. Vuorelainen, O., The temperature field produced in the ground by a heated slab laid direct on ground, and the heat flow from slab to ground, *Acta Polytech. Scandinavica* no. 278, (Civil Engng. and Building Construction Series no. 8), 60 pp., 1960.

The mathematical equations of the temperature distribution field produced by a warm floor slab in the underlying homogeneous, isotropic ground have been solved and the heat quantities flowing from the slab to the ground, in the stationary state, have been calculated for slabs of rectangular form, of the shape of a narrow strip, and of circular shape. In the solutions those boundary conditions have been found which are most appropriate for use in cases occurring in actual practice.

Nonstationary temperature fields for which the surface temperature of the slab or the thermal flow from the slab to the ground is known as a function of time are presented for a slab having the shape of a narrow strip. The equations can also be employed to calculate the field, varying with time, which exists in the vertical section in the middle of an elongated, rectangular slab. Fourier's and Hankel's transformations have been employed in the solutions and the unknown functions involved have been found by means of a systematic procedure, e.g., as a solutions of dual integral equations.

From author's summary

Reviewer believes this work would be much more useful if numerical (or graphical) results were given for the rectangular shapes, and a comparison made with some of the approximate methods referred to in the text.

R. V. Dunkle, Australia

Book—4519. Rymsha, V. A., Investigation of ice formation in rivers and storage reservoirs [Ledovyie issledovaniia na rekakh i vodokhranilishchakh], Leningrad, Gidrometeoizdat, 1959, 156 pp. $1.96.

A lucid treatise covers phenomena of ice formation in flowing and stagnant water; methods of heat-loss determination from the open surface; thermic behavior of rivers and reservoirs during winter period; devices for temperature measurement of water and ice; ice phenomena during a freezing period and before an ice-break in spring; physical properties of ice and frazil. This well-prepared booklet deals with a very important problem for hydraulic engineering and reflects wide experience obtained in conditions of Russia. An English translation of this monograph should be very appropriate.

S. Kolupaila, USA

4520. Makkaveev, V. M., Structure of turbulent river-bed flows (in Russian), *Trudi Leningrad In-ta Inzh. Vodn. Transp.* no. 25, 3–8, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10126.

An investigation is made of the possibility of establishing a linkage between the fluctuations in the velocity of a turbulent flow at its thickest point and the deformations of the free surface. The proposal is made to develop the suggested special scheme to include the propagation of bottom layers in a deforming river-bed.

V. N. Goncharov

Courtesy Referativnyi Zhurnal, USSR

4521. Yih, C.-S., A transformation for non-homentropic flows with an application to large-amplitude motion in the atmosphere, *J. Fluid Mech.* **9,** 1, 68-80, Sept. 1960.

Transformation is applied to the equations of motion and continuity of an inviscid nonhomentropic flow in the absence of gravity so that they are amenable to solution as an "associated" homentropic flow having the same flow pattern and sustaining the same lift and drag forces. Comparisons are made with Crocco's stream function and method is applied to curved shock waves with upstream nonhomentropic flow.

Equations are extended to include gravity and the particular cases of swirling axisymmetric flows, and gravity waves in air in lee of mountains. E. L. Houghton, England

4522. Krasovskii, V. I., Results of scientific investigations made by Soviet sputniks and cosmic rockets, *ARS J.* **30,** 1, 27-33, Mar. 1960.

Article summarizes data relative to measurements of the upper atmosphere and "cosmic" space. It includes information relative to cosmic radiation, interplanetary dust and magnetic fields at varying distances from the earth. Sputnik data include measurements of cosmic radiation as a function of altitude and latitude 55 ° to 65 ° N. S. Lampert, USA

4523. Ainsworth, J. E., Fox, D. F., and Lagow, H. E., Measurement of upper-atmosphere structure by means of the pitot-static tube, NASA TN D-670, 47 pp., Feb. 1961.

Profiles of atmospheric pressure, density, and temperature for the region 20 to 110 km above Fort Churchill, Canada, were computed from the data obtained during a single fall-day flight. The temperature profile had a major maximum at 58 km, a secondary maximum at 82 km, and several minor maxima in the region from 20 to 55 km. During the same flight a horizontal-wind profile was obtained in the region from 80 to 115 km. The average wind speed was 125 m/sec, and the maximum was 250 m/sec at 118 km. The instrumentation, method of measurement, and results are discussed in detail. Methods are prescribed for using the pitot-static tube for accurate synoptic measurements of atmospheric structure from 20 to 80 km and for measuring possible semi-diurnal pressure variations in the same region. From authors' summary

4524. Khorosheva, V. V., The influence of atmospheric pressure on inclinations of the earth's surface (in Russian), *Izv. Akad. Nauk SSSR Ser. Geoiz.* no. 1, 131-135, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4284.

In order to determine the inclinations (slopes) and displacements of the Earth's surface under the action of cyclones and anti-cyclones, the known solution is applied of the problem of an elastic semi-space, loaded on the bounding plane by a normal force uniformly distributed over a circular area. For the case of simultaneous action of a plurality of cyclones and anti-cyclones, the resulting, summated inclination of the Earth's surface is determined. This solution is applied to the evaluation of factual observations. M. V. Malyshev

Courtesy Referativnyi Zhurnal, USSR

4525. Shupyanskii, A. B., Measurements of the velocities and form of water drops and of rain (in Russian), *Trudî Tsentr. Aerol. Observ.* no. 22, 73-88, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5283.

Measurements were made of the velocity of fall of drops, having radii of from 0.3 to 3 mm, by means of the induction method and simultaneous photography, the source of illumination being an impulse lamp. The exposure took place in transient light with the impulse lamp triggered by a relay which operated when current impulses appeared in one of the induction rings through which the descending drop had to fall. The height of the fall varied from 0.5 to 15 m. The drops on release from a capillary tube were electrically charged with the aid of an induction ring to which a potential was led. Values were obtained for the velocity of fall of drops of different dimensions and computations were carried out for the magnitude of the Reynolds number and the resistance coefficient. The latter showed little variation in magnitude for drops of radii between 1 and 3 mm. Noticeable deformation in the drops during the fall took place when the drops had radii exceeding 1.5 mm. Analogous measurements were also made for raindrops. The values for $d \geqslant 0.7$ mm are satisfactorily described by means of the empirical formula $v = \lg 8320\, d^{0.85}$, where v is the velocity of fall of the drop in m/sec while d is the diameter of the drop in mm. N. S. Shishkin

Courtesy Referativnyi Zhurnal, USSR

Naval Architecture and Marine Engineering

(See also Revs. 4312, 4331)

4526. Kaplan, P., and Henry, C. J., A study of the hydroelastic instabilities of supercavitating hydrofoils, *J. Ship Res.* **4,** 3, 28-38, Dec. 1960.

Theoretical study deals with two-dimensional case of rigid foils on elastic support; static (torsional divergence) and dynamic (bending torsion flutter) instabilities are investigated using hydrodynamic results obtained mainly by M. Tulin and L. G. Woods. The influence of variations of several problem parameters is determined mainly for zero cavitation number σ; in addition conditions at low σ are hypothesized. Contrary to fully wetted flow, both static and dynamic instabilities may occur at relevant density ratios. Experimental check is recommended because of hypotheses assumed. G. P. Weinblum, Germany

4527. Wilson, B. W., Mooring of ships exposed to waves, Texas A & M, Dept. Oceanogr. and Meteor. TR 204-2, 65 pp., Nov. 1960.

4528. Arndt, B., Kastner, S., and Roden, S., Stability testing of sailing boat "Gorch Fock" (in German), *Schiffstechnik* **7,** 39, 177-190, Nov. 1960.

4529. Vossers, G., Fundamentals of the behaviour of ships in waves: Part 2, Ship motions, *Inter. Shipbldg. Prog.* **6,** 64, 538-555, Dec. 1959; **7,** 65, 28-46, Jan. 1960; **7,** 66, 77-95, Feb. 1960; **7,** 67, 121-135, Mar. 1960; **7,** 68, 172-185, Apr. 1960; **7,** 69, 221-227, May 1960; **7,** 70, 261-279, June 1960.

Differential equations for periodic motion of a floating body are derived in most general form. Dependence of coefficients on frequency is examined. Special sections are dedicated to added mass and moment of inertia; damping; restoring forces and moments; exciting forces and moments; coupling; and to nonlinear effects. A further section on ship motions in irregular seas elucidates the contribution of above terms to the "response function" governing the correlation between the spectra of existing wave system and ship motions generated. 130 diagrams and 200 papers quoted! K. Eggers, Germany

4530. Lugovskoi, V. V., Approximation methods of nonlinear mechanics, in the theory of the rolling of ships in waves (in Russian), *Trudî Leningrad Korablestroit. In-ta* no. 22, 65-77, 1958; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 3800.

A purely methodical problem is put forward for solution, concerning the analysis of the known methods of nonlinear mechanics as applied to a second-order differential equation, to which, in the particular case, the problem of the rolling of a ship can be re-

duced, in A. N. Krylov's formulation. A brief account is given of the methods of calculation involved: the method of the harmonic balance; Van-der-Pol's method; the energy method; the Ritz-Galerkin method; the method of the small parameter in the formulation of Poincare' and Liapunov; the Malkin method; and the Rauscher method. The paper is of no scientific value, but could serve as an aid for engineers. N. N. Moiseev

Courtesy Referativnyi Zhurnal, USSR

4531. Oversmith, R. H., Cavitation of axially symmetrical bodies in accelerating motion, *J. Ship Res.* **4,** 3, 39-47, Dec. 1960.

A dimensional analysis is made and test results for three nose forms are presented. Author concludes "...drag rise with acceleration will be significant for a high-speed underwater vehicle only at relatively low velocities, and should have little effect on the over-all performance of the craft;...cavitation may occur at relatively low velocities in accelerating...motion compared to (the same velocities in) steady state...."

From author's summary by P. J. Mullan, USA

4532. Fednievskii, K. K., A proof of the steady-state hypothesis for determining the hydrodynamic forces and moments acting on a ship moving in a horizontal plane (in Russian), *Trudi Naucbno-Tekbn. Ov-a Sudostroit. Prom-sti* **1,** 2, 171-181, 1957; *Ref. Zh. Mekb.* no. 4, 1959, Rev. 3798.

A comparison of the rotational, derivative forces of an inertial character with the forces of viscosity is used to demonstrate the possibility of neglecting the components of the forces and moments due to viscosity, and associated with the variability of the vortex train behind rectangular and triangular profiles of small aspect ratio, performing small angular harmonic oscillations. The result is applied to the case of a ship moving in a horizontal plane. This hypothesis is used to set up new expressions for the rotatory, derivative forces of inertial character in the unsteady motion of bodies of small aspect ratio, which coincide for $\lambda \longrightarrow 0$ with results already obtained. It is demonstrated that the hypothesis of the steady state can likewise be applied to the analysis of the aperiodic motions of a ship in a horizontal plane.

A. N. Bazilevskii

Courtesy Referativnyi Zhurnal, USSR

Friction, Lubrication and Wear

(See also Revs. 4183, 4184, 4190, 4217)

4533. Daniel, T. B., Surface phenomena and friction, First Symposium on Surface Effects on Spacecraft Materials, Palo Alto, Calif., May 12-13; 1959; New York, John Wiley & Sons, Inc., 1960, 307-318.

Theories are discussed which attempt to explain friction in terms of micro-physical phenomena occurring in the interface between sliding objects. Friction studies are reviewed with emphasis on incomplete experiments with the graphite and MoS_2 systems.

F. Forscher, USA

4534. Epifanov, G. I., On the role of adhesion in the formation of the frictional force on clean surfaces, *Soviet Phys.-Doklady* **5,** 4, 865-867, Jan./Feb. 1961. (Translation of *Dokladl Akad. Nauk SSSR (N.S.)* **133,** 2, 349-351, July 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

4535. Zaid, M., and Tolins, I. S., The measurement of very low frictional torques in rotating equipment, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 382-386, Nov. 1960.

The frictional torque in such devices as air bearings is measured and resolved into coulomb and viscous friction components by analysis of the observed deceleration during free rotation. Two methods of analysis are given. Both are based on the differential equation of motion of the rotating system and give results that are in satisfactory mutual agreement. In the first method an integration is obtained from which a phase plane diagram is derived in which the friction components are read off as slopes and intercepts. To carry out the analysis measurements of angular displacement, time and velocity are required. The second method proceeds from a different integration of the equation of motion, is more complex, but requires measurements of only angular displacement and velocity. Reviewer believes that the general applicability of these methods will make them of interest to many workers in the fields of friction and lubrication.

J. W. Givens, USA

4536. Kosterin, Yu. I., and Sidorenko, G. S., Static frictional characteristics of nonmetallic materials (in Russian), *Vestnik Masb.* **40,** 8, 38-41, Aug. 1960.

In the first part of the paper, authors consider a number of expressions for determining the frictional force developed between two nonmetallic materials as well as between nonmetallic materials operating together with metals, based upon studies published in Russian literature. They describe their own experiments conducted on the device, in which the relative velocities of tested materials were between 0.05 and 3.0 m/sec, and the load between 0.6 and 10 kg.

Combinations of the following materials were tested: plexiglass—steel; plexiglass—copper; frictional material 6-KH-1—steel at room temperature; material 6-KH-1—steel at 170°C; and lead-copper. The relationship between the frictional force and the duration of motionless contact, as well as the relationship between the static frictional force and the load were investigated experimentally. Also, the relationship between the frictional force and the temperature was studied.

The study shows a good correlation between the experimental data and the data calculated from the formula

$$T = T_\infty (1 - e^{-\delta t^\rho})^\nu$$

where

T = frictional force for dry friction
T_∞ = frictional force after a contact of infinite duration
δ and ρ = constants depending upon the materials
t = time of contact

$$T_\infty = \alpha A_a^{\frac{m}{\nu+m}} b^{\frac{m}{\nu+m}} \left(\frac{\nu+m}{B}\right)^{\frac{\nu}{\nu+m}} N^{\frac{\nu}{\nu+m}}$$

α = constant characterizing the interactions of the surfaces of the materials
A_a = nominal area of contact
b and ν = constants characterizing the geometry of the surfaces and depending upon the method of machining
B and m = constants which depend upon compression of materials.

Two tables in the text gave the data about some of the constants in this expression. Three diagrams present the obtained experimental data in graphical form. E. I. Radzimovsky, USA

4537. Rouverol, W. S., and Tanner, R. I., A brief examination of factors affecting tractive friction coefficients of spheres rolling on flat plates, *ASLE Trans.* **3,** 1, 11-17, Apr. 1960.

A study was made of factors affecting the operation of a new variable speed transmission in which torque is transmitted by moving balls pressed between counter rotating loaded disks [*Engineering* **183,** 4762, p. 754, June 14, 1957]. Using a test machine in which a single steel ball was pressed between two disks, it

was found that the coefficient of tractive friction, which is the ratio of tractive to normal force, was found to be relatively independent of normal load and rolling velocity. Tractive friction coefficient increased as the ball diameter decreased and varied considerably with lubricant properties. High-viscosity gear oils in copious supply affected it unfavorably, i.e. decreased it. Low-viscosity fluids gave values of the order of 0.08. Special types of fluids such as chlorinated biphenyl, polybutene and oils containing extreme pressure additives were also studied. S.A.E. 20 oil was the most satisfactory. J. W. Givens, USA

4538. Johnson, K. L., A note on the influence of elastic compliance on sliding friction in ball bearings, *ASME Trans.* **82**D (*J. Basic Engng.*), 4, 899–900, Dec. 1960.

Author reviews effect of contact friction, slip pattern, and elasticity on ball spin in a ball bearing. Experiments show interestingly that no slip exists at leading edge of a ball which is also spinning during slow speed centrifugal rolling in a bearing race. The variation in slip in the ball-race contact gives a lower frictional spinning moment than expected for a rotationally symmetrical case. At high speed, hydrodynamic action is speculated to offer further complications. E. R. Booser, USA

4539. Peterson, M. B., Florek, J. J., and Lee, R. E., Sliding characteristics of metals at high temperatures, *ASLE Trans.* **3**, 1, 101–109, Apr. 1960.

Friction tests were run with a number of metals sliding against themselves. With iron, copper, nickel, molybdenum and chromium, high friction and considerable surface damage resulted until a particular transition temperature was reached; above this temperature a considerable improvement in the sliding characteristics was observed. The transition temperature for the various metals was as follows: iron, 100–200 F; copper, 400–500 F; nickel, 1200–1400 F; molybdenum, 800–900 F; and chromium, 800–1100 F. This improvement was associated with the generation of oxide in the sliding track. When tested as lubricants a number of the metal oxides were effective in reducing friction and preventing surface damage. Since the softer oxides were the more effective it was hypothesized that metals which form these oxides would have the better sliding characteristics at high temperatures. The friction characteristics of alloys were compared with the frictional characteristics of the major constituents.

From authors' summary

4540. Seifert, A., Cylinder and piston ring wear on diesel engines for tractors (in German), *Wear* **3**, 6, 426–439, Nov./Dec. 1960.

4541. Bartel, A. A., Gear lubrication: Part 1, *ZVDI* **103**, 6, 251–264, Feb. 1961.

The way leading to the present level of lubrication practice has been a long one. It may be said that, in principle, satisfactory solutions have been produced for the oil, grease and gas lubrication of small gearings. The pressure behavior of the viscosity (apart from the temperature behavior) plays a vital role with regard to large-size gearings and those running at high speeds. Extensive tests have already done much to elucidate the physical procedures and have produced an abundance of useful data.

From author's summary

4542. Yen, K. T., On the compressibility effects of the lubricant for two-dimensional slider bearings, *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 609–612, Dec. 1960.

An analytical study of compressibility effects on characteristics of two-dimensional slider bearings is made. Analysis indicates that no optimum film shape (such as stepped films for incompressible flow) exists for compressible flow. Compressible flow numerical analysis for stepped films indicates that nonexistence of optimum film shape is due to nonlinear nature of compressible problem.

From author's summary by W. J. Anderson, USA

4543. Dowson, D., and Higginson, G. R., The effect of material properties on the lubrication of elastic rollers, *J. Mech. Engng. Sci.* **2**, 3, 188–194, Sept. 1960.

An analytical study, for isothermal flow, gives the pressure distributions and film shapes for five cases to illustrate the influence of load, speed, and materials. Maximum principal stresses are shown for two solutions. For usual materials, speeds, and loads, the outlet pressure peak predicted by Grubin is always present. The magnitude of this peak may exceed the maximum Hertzian pressure. The outlet pressure peak does not vary much with load, but varies markedly with speed and materials. The centerline film thickness does not vary much with load, but varies significantly with material properties and the product of speed and inlet viscosity. The stress concentrations produced near the surface by the outlet pressure peak may have a significant effect on the fatigue life of rolling elements.

R. C. Binder, USA

4544. Feng, I.-M., Pyrolysis of zinc dialkyl phosphorodithioate and boundary lubrication, *Wear* **3**, 4, 309–311 (Brief Note), July/Aug. 1960.

Zinc dialkyl phosphorodithioate is commonly added to lubricating oils as a mild E.P. (extreme pressure) agent. The thermal decomposition of this substance was studied by heating in nitrogen at 176–177° C. About 11.6% volatile substances were evolved, which proved to be H_2S olefins. The nonvolatile residue thickened and eventually solidified. Author postulates that the evolution of H_2S under frictional heating explained the E.P. action of the compound. Under more severe conditions polymerization *in situ* provides a protective surface layer. It is necessary to stress that the suggested mechanisms derived from experiments *in vitro* may not be fully applicable when decomposition takes place in contact with strained metal surfaces.

L. Grunberg, Scotland

4545. Fedor, J. V., A Sommerfeld solution for finite bearings with circumferential grooves, *ASME Trans.* **82**D (*J. Basic Engng.*), 2, 321–326, June 1960.

The paper suggests an analytical solution of Reynolds three-dimensional equation for journal bearings with circumferential groove. Author presents the pressure function of such a bearing in an expression consisting of three terms. The first term of this expression is "the circumferential source function," the second term is the pressure equation for an infinite full journal bearing, and the third term represents a "correction function," which takes into account the finiteness of the bearing (influence of the end leakage). By expanding the pressure equation for an infinite bearing in a Fourier series and using the boundary conditions at the end of the bearing (pressure = zero), the solution for the third term in the equation and, therefore, for the entire function is found. The method is illustrated by using it to determine the bearing characteristics such as load capacity, coefficient of friction, and the axial flow in the bearing.

The results of calculating the load capacity, using the solution obtained, are compared with results of approximate solutions by Muskat and Morgan, obtained for conditions when the attitude (the relative eccentricity) is less than 0.6, and good agreement is found.

The suggested solution is applicable to the full journal bearings for which Sommerfeld boundary conditions may be used. These conditions are based upon the assumption that the journal

in the bearing is completely surrounded by an uninterrupted oil film, so that the pressure is a continuous recurrent function, and the pressure at any angle θ is equal to the pressure at angle $(\theta + 2\pi)$.

E. I. Radzimovsky, USA

4546. Kingsbury, E. P., The heat of adsorption of a boundary lubricant, *ASLE Trans.* **3**, 1, 30–33, Apr. 1960.

In a previous paper the author has shown that the heat of adsorption of a boundary lubricant determines its frictional characteristics [*J. Appl. Phys.* **29**, p. 888, 1958; AMR **11** (1958), Rev. 5258]. This paper extends this theory to include wear. A characteristic temperature is calculated at which the film defect increases most rapidly and marks the transition from good to poor sliding. This is shown to vary from −35°C for a silicone oil to 550° for molten zinc. This shows the possibility of using molten metals as lubricants in this temperature range.

J. W. Givens, USA

4547. Peale, L. F., Messina, J., Ackerman, B., Sasin, R., and Swern, D., Evaluation of long-chain phosphorus compounds as lubricity additives, *ASLE Trans.* **3**, 1, 48–54, Apr. 1960.

4548. Sabroff, A. M., and Frost, P. D., A comparison of lubricants and coatings for cold extruding titanium, *ASLE Trans.* **3**, 1, 61–68, Apr. 1960.

4549. Mahoney, C. L., Barnum, E. R., Kerlin, W. W., and Sax, K. J., Meta-linked polyphenyl ethers as high-temperature radiation-resistant lubricants, *ASLE Trans.* **3**, 1, 83–92, Apr. 1960.

4550. Darling, S. M., and Musselman, J. M., The contribution of polymers to oil properties important to engine lubrication, *ASLE Trans.* **3**, 1, 134–141, Apr. 1960.

4551. Baskey, R. H., An investigation of seal materials for high-temperature application, *ASLE Trans.* **3**, 1, 116–123, Apr. 1960.

Novel rotating seal materials were developed by powder metallurgy techniques for potential aircraft applications at high speeds and high temperatures.

A systematic wear study without lubrication included several commercially available materials and the following types of experimental materials: (a) pure refractory hard metals, (b) binary alloys of refractory hard metals bonded with nickel, and (c) ternary alloys of refractory hard metals bonded with nickel and infiltrated with silver.

Two ternary alloys, containing nickel bonded WB or CrN and infiltrated with silver, showed superior wear qualities against either tool steel or a nickel–chromium–iron alloy at a sliding speed of 29,000 ft/min under a 14 lb/in^2 load and at ambient temperatures as high as 1300 F. A commercial titanium carbide composition showed excellent wear characteristics in contact with an identical composition at a sliding speed of 14,000 ft/min under a 16 lb/in^2 load and at an ambient temperature of 1050 F.

From author's summary

4552. Baber, B. B., Lawler, C. W., Smith, H. R., Beane, G. A., and Ku, P. M., Gear lubrication in inert gas atmospheres, *ASLE Trans.* **3**, 1, 142–148, Apr. 1960.

An investigation was made of the effect of inert gas atmospheres on the gear load-carrying capacity of lubricants. The experiments were performed in two types of gear test machines, using case-hardened AMS-6260 steel test gears. It was found that two mineral oils (a solvent-extracted turbine oil base stock and a USP grade white mineral oil), as well as the same oils fortified with different extreme-pressure additives, all exhibited a decided increase in load-carrying capacity when the gears were operated in an atmosphere of nitrogen or argon instead of air. On the other hand, the response of synthetic lubricants was not found to be necessarily similar. In fact, only one of the six synthetic fluids tested showed any significant increase in load-carrying capacity when the gears were operated in a nitrogen atmosphere.

From authors' summary

4553. Nigam, S. D., and Gupta, S. C., A boundary value problem for an elliptic equation in the theory of gas-lubricated bearings, *Appl. Scient. Res.* (*A*) **9**, 6, 463–464 (Letter to the Editor), 1960.

4554. Newman, A. D., Extreme pressure lubricants for marine gears, *Proc. Instn. Mech. Engrs.* **174**, 5, 241–269, 1960.

4555. Ku, P. M., and Baber, B. B., The effect of lubricants on gear tooth scuffing, *ASLE Trans.* **2**, 2, 184–194, Oct. 1959.

The effects of lubricants, test equipment variables and operating variables on the scuff-limited load in gear lubrication were studied by means of the Ryder gear machine. Tests were made with mineral and synthetic oils. The scuff-limited load increased rapidly with viscosity, thereby the importance of the hydrodynamic factor was established. The scuff-limited load was also increased by oxidative deterioration. The effects of pressure angle of the gears, gear material, and surface hardness were not large. The effect of speed on load-carrying capacity is in agreement with the results of Borsoff [*ASME Trans.* **81** D, 1, 79–93, 1959; AMR **12** (1959), Rev. 3188], who showed that load-carrying capacity passed through a minimum as speed increases. The load-carrying capacity also increased with lubricant flow rate and with feed to the meshing side of the gears. Of particular interest is the result that the scuff-limited load could be markedly increased by operation in inert atmospheres, such as nitrogen or argon. Reviewer believes that this is a comprehensive paper and will be of interest to lubrication engineers and to gear designers.

J. W. Givens, USA

4556. Cameron, A., A theory of boundary lubrication, *ASLE Trans.* **2**, 2, 195–198, Oct. 1959.

Starting with the premise that boundary friction is primarily due to the van der Waals forces between adsorbed lubricant molecules, rather than to welding and tearing of surfaces, a calculation of the coefficient of friction between layers of paraffins is made. The attractive and repulsive forces for hydrocarbon chains were calculated by the method of Muller [*Proc. Roy. Soc. Lond.* (*A*) **154**, p. 624, 1936; ibid. **178**, p. 227, 1941]. The calculation leads to value of the coefficient of friction of about 0.2, which agrees within orders of magnitude with experimental values.

J. W. Givens, USA

4557. Cosgrove, S. L., Sibley, L. B., and Allen, C. M., Evaluation of dry powdered lubricants at 1000 F in a modified four-ball wear machine, *ASLE Trans.* **2**, 2, 217–224, Oct. 1959.

The friction and wear behavior of selected solid lubricants and wear specimens has been studied in a modified four-ball wear machine. Data were obtained at 1000 F for sliding speeds of 120 and 700 ft per min under initial Hertzian contact stresses of about 250,000 psi. Conventional solid lubricant material such as lead oxide (litharge), molybdenum disulfide, and graphite were evaluated. A new dry solid lubricant, metal-free phthalocyanine, was studied and was usually found to have superior lubricating ability. Wear specimens of AISI Type M-1 tool steel and titanium carbide-nickel-molybdenum cermet showed the most promise for high-temperature sliding systems. Initial coefficients of friction less than 0.1 were obtained with combinations of these materials.

From authors' summary

4558. Peterson, M. B., Murray, S. F., and Florek, J. J., Consideration of lubricants for temperature above 1000 F, *ASLE Trans.* **2,** 2, 225–234, Oct. 1959.

A study has been made of the various solid and liquid lubricants which could be considered for use to temperatures above 540 C. This study consisted of a literature survey to isolate temperature-stable, low shear strength compounds, and some experimental work to evaluate the most promising materials. Many of these compounds were effective at high temperatures and the results can explain the compatibility of certain metals at high temperatures. These compounds were not effective at lower temperatures. A more detailed study was made of the sliding characteristics of metals when lubricated with silver and molten boric oxide. The results show that, for silver, a number of factors such as choice of bearing materials, strength, and impurities affect the frictional behavior. Boric oxide, under conditions of boundary lubrication, showed a large increase in friction when the viscosity increased above 2400 poise. Variations in the friction could be explained by independent variations of viscosity and shear area.

No lubricants were isolated which could be considered completely effective for a temperature range from 27 C to above 540 C. The choice of bearing materials and designs will have to be made to minimize these deficiencies.

From authors' summary

4559. Backers, F. T., A magnetic journal bearing, *Philips Tech. Rev.* **22,** 7, 232–238, 1960/61.

Supporting a rotating shaft in such a way that no material contact is made with the shaft can be of importance for technical applications. Cases in point are where the friction or wear must be particularly small where contamination by lubricants is inadmissible or where the lubricating oil would decompose under the influence of radiation (in a nuclear reactor, for example). Author considers the theory of a shaft which is held in suspension by magnetic fields, and compares the theory with the results of measurements which have been made on "magnetic bearings" of various dimensions. From author's summary

4560. Holmes, R., The vibration of a rigid shaft on short sleeve bearings, *J. Mech. Engng. Sci.* **2,** 4, 337–341, Dec. 1960.

Author treats whirl of shaft supported between nonpressurized journal bearings. Flexible shaft is to be considered in later paper. Making Ocvirk's assumptions that length-to-radius ratio is less than 2 and oil is present only in convergent space, author deduces dynamic oil-film forces on journal in terms of linearized, velocity and displacement-dependent quantities, having started with appropriate form of Reynolds equation. Constant viscosity is assumed. Frequency response is obtained in dimensionless form, and stability conditions deduced from it and represented graphically. Plot indicates absence of instability for eccentricity ratios above 0.8, which was shown earlier by Hori for infinitely long bearings. Whirl predictions are better for decreasing displacement ratios. Linearization makes them conservative since nonlinearities tend to reduce amplitudes. Well-known rule that initial whirl frequency is approximately half running speed is illustrated, but possibility of lower frequency whirl is also confirmed. T. Ranov, USA

4561. Hull, E. H., Control of oil whip frequency ratio, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 894–898, Dec. 1960.

Author inserts an intermediate rotatable sleeve in the clearance space of a conventional full journal bearing. Experiments show that driving externally this sleeve changes the velocity distribution in the oil film and hence the speed at which whip occurs. It is suggested that this technique can be applied practically to control the oil whip frequency ratio.

A. A. Raimondi, USA

4562. Laub, J. H. Elastic orifices for gas bearings, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 980–982 (Tech. Briefs), Dec. 1960.

Author claims two advantages over conventional fixed-diameter type orifices: (a) lower flow requirements and (b) improved stiffness. Test data show these gains with some sacrifice in film thickness. Orifices were made from an elastomer which deformed with supply pressure so as to reduce flow area.

A. A. Raimondi, USA

4563. Northrup, G. R., Solution for a high speed bearing problem, *Wear* **3,** 6, 413–420, Nov./Dec. 1960.

4564. Powell, D. L., and Barton, H. R., Analytical study of surface loading and sliding velocity of automotive hypoid gears, *ASLE Trans.* **2,** 2, 173–183, Oct. 1959.

An analysis has been made that translates the prescribed operating procedures in the CRC L19, L20, L37 and L42 hypoid gear tests into actual gear loads and sliding velocities. Similar calculations were carried out for the results of highway truck gear tests previously reported by the same authors (SAE National Fuels and Lubricants Meeting, November 7, 1957). Thereby sliding velocities and gear loads could be correlated with highway conditions. The sliding velocities and directions for two automobile and two truck drive systems are shown. The trends in hypoid gear design for passenger cars since 1946 are analyzed and it is shown that pinion offset tends to increase and gear ratio to decrease. These two trends are of opposite effect on sliding velocity, hence this parameter tends to remain constant.

Reviewer believes that this paper will be of interest to many workers concerned with hypoid gears because it makes it possible to convert operating variables into gear tooth face environment.

J. W. Givens, USA

4565. Biderman, V. L., Calculation of the critical rolling speed of a pneumatic tire (in Russian), *Trudi Nauk-i. In-ta Shin. Promsti* no. 3, 64–75, 1957; *Ref. Zh. Mekh.* no. 4, 1959, Rev. 4529.

At some particular rolling speed of a pneumatic tire corrugations appear on its surface (tread) moving at the speed of rolling, with reference to the material of the tire. This rolling speed of the tire is a critical value. The life of tires running at a critical speed is measured in minutes. In order to raise the critical speed value and lengthen the life of the tires of high-speed (racing) motor cars, the internal tire pressure is increased, and the mass of the tire decreased. Formulas and curves are given for determining the relationships between critical rolling speed, internal tire pressure, thread angle of the cord fabric, and rigidity of the rubber. The allowable service speed of a tire is determined by multiplying the calculated critical speed by a factor of safety less than unity. K. P. Korolev

Courtesy Referativnyi Zhurnal, USSR

Letter to the Editor

4566. Re: AMR **14**(1961), Rev. 2309; **Schjodt, R., Difference equations for physical and technical problems** (in English), Tekn. Str., Norges Tekniske Hogskole, Oslo, Norway no. 21 N, 12 pp., 1960.

First sentence of review should have read: "Paper shows that in certain cases, particularly near irregular boundaries, the finite difference equations obtained from a differential equation do not correspond exactly to those obtained by writing the basic equilibrium conditions for finite elements.

The editors regret this error.

Books Received for Review

AMBARTSUMYAN, S. A., Theory of anisotropic shells [Teoriya anizotropnykh obolochek], Moskva, Fizmatgiz, 1961, 384 pp. 1 r 43 K.

Analysis of strength, Vol. 6 [Raschetyna prochnost'], Moskva, Mashgiz, 1960, 313 pp. 11 r 18 K.

BERMAN, A. I., The physical principles of astronautics—fundamentals of dynamical astronomy and space flight, New York, John Wiley & Sons, Inc., 1961, xv + 350 pp. $9.25.

BIGNOLI, A. J., and MARTOCCIA, R. F., El metodo de los puntos fijos-atlas, Buenos Aires, Centro Estudiantes de Ingenieria, 1960, 142 pp. and 25 lamina.

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DEDANI, A., advisory editor, Glass fibre reinforced plastics, New York, Interscience Publishers, Inc., 1960, viii + 296 pp. $9.75.

Fluid meters—their theory and application, 5th ed., New York, American Society of Mechanical Engineers, 1959, 203 pp. $8.

GEBHART, B., Heat transfer, New York, McGraw-Hill Book Co., 1961, x + 454 pp. $10.75.

Handbook of thermophysical properties of solid materials, Vol. I: Elements, New York, Pergamon Press, 1961, vi + 758 pp. ($90. the 5-volume set)

HEARMON, R. F. S., An introduction to applied anisotropic elasticity, New York, Oxford University Press, 1961, viii + 136 pp. $5.60.

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MYERSON, A. L., and HARRISON, A. C., edited by, Physical chemistry in aerodynamics and space flight (Proceedings of the Conference on Physical Chemistry in Aerodynamics and Space Flight, University of Pennsylvania, 1-3 Sept. 1959), New York, Pergamon Press, 1961, viii + 288 pp.

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Symposium on Compressor Stall, Surge and System Response, Joint Conference ASME Gas Turbine Power and Hydraulic Divisions, Houston, Texas, Mar. 6–9, 1960; New York, American Society of Mechanical Engineers, 1960, 56 pp. (Paperbound)

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